Solutions Manual

Fundamentals of Engineering Electromagnetics

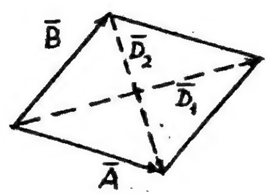
DAVID K. CHENG

CENTENNIAL PROFESSOR EMERITUS, SYRACUSE UNIVERSITY

Chapter 2

Vector Analysis

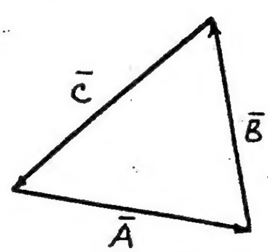
P. 2-1 Denoting the diagonals of the rhombus by $\bar{D}_1$ and $\bar{D}_2$, we have:



(a) $\bar{D}_1 = \bar{A} + \bar{B}$,
$\bar{D}_2 = \bar{A} - \bar{B}$.

(b) $\bar{D}_1 \cdot \bar{D}_2 = (\bar{A}+\bar{B})\cdot(\bar{A}-\bar{B})$
$= \bar{A}\cdot\bar{A} - \bar{B}\cdot\bar{B} = 0$,
since $|\bar{A}| = |\bar{B}|$.
Thus, $\bar{D}_1 \perp \bar{D}_2$.

P. 2-2



$\bar{A} + \bar{B} + \bar{C} = 0$.

$\bar{A}\times$: $\bar{A}\times\bar{B} = \bar{C}\times\bar{A}$.

$\bar{C}\times$: $\bar{C}\times\bar{A} = \bar{B}\times\bar{C}$.

$\bar{B}\times$: $\bar{B}\times\bar{C} = \bar{A}\times\bar{B}$.

Magnitude relations:
$AB\sin\theta_{AB} = CA\sin\theta_{CA} = BC\sin\theta_{BC}$.

Hence,
$$\frac{A}{\sin\theta_{BC}} = \frac{B}{\sin\theta_{CA}} = \frac{C}{\sin\theta_{AB}}.\quad \text{(Law of sines.)}$$

P. 2-3 a) $\bar{a}_B = \dfrac{\bar{a}_x 4 - \bar{a}_y 6 + \bar{a}_z 12}{\sqrt{4^2+6^2+12^2}} = \bar{a}_x\dfrac{2}{7} - \bar{a}_y\dfrac{3}{7} + \bar{a}_z\dfrac{6}{7}$.

b) $\bar{B}-\bar{A} = -\bar{a}_x 2 - \bar{a}_y 8 + \bar{a}_z 15$, $\quad |\bar{B}-\bar{A}| = \sqrt{2^2+8^2+15^2} = 17.1$.

c) $\bar{A}\cdot\bar{a}_B = 6\times\frac{2}{7} - 2\times\frac{3}{7} - 3\times\frac{6}{7} = -17.1$.

d) $\bar{B}\cdot\bar{A} = 24 - 12 - 36 = -24$.

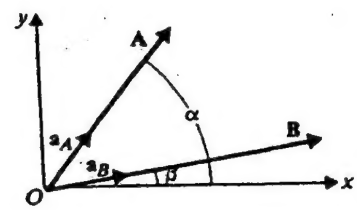
e) $\bar{B}\cdot\bar{a}_A = \dfrac{\bar{B}\cdot\bar{A}}{|\bar{A}|} = \dfrac{-24}{\sqrt{6^2+2^2+3^2}} = -\dfrac{24}{7} = -3.43$.

f) $\cos\theta_{AB} = \dfrac{\bar{B}\cdot\bar{A}}{BA} = \dfrac{-24}{14\times 7} = -0.245$, $\quad \theta_{AB} = 180^\circ - 75.8^\circ = 104.2^\circ$

g)
$$\bar{A}\times\bar{C} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 6 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = -\bar{a}_x 4 - \bar{a}_y 3 - \bar{a}_z 10$$

h) $\bar{A}\cdot(\bar{B}\times\bar{C}) = (\bar{A}\times\bar{B})\cdot\bar{C} = -(\bar{A}\times\bar{C})\cdot\bar{B} = -[(-4)4+(-3)(-6)+(-10)12] = -118.$

P.2-4



$$\bar{a}_A = \bar{a}_x\cos\alpha + \bar{a}_y\sin\alpha,$$
$$\bar{a}_B = \bar{a}_x\cos\beta + \bar{a}_y\sin\beta.$$

a) $\bar{a}_A\cdot\bar{a}_B = \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$

b)
$$\bar{a}_B\times\bar{a}_A = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = \bar{a}_z(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$$
$$= \bar{a}_z\sin(\alpha-\beta).$$
$$\therefore\ \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

P.2-5 a) $\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = -\bar{a}_x 4 + \bar{a}_y + \bar{a}_z 3,$
$\overrightarrow{P_2P_3} = \overrightarrow{OP_3} - \overrightarrow{OP_2} = \bar{a}_x 6 - \bar{a}_y 5 + \bar{a}_z,$
$\overrightarrow{P_1P_3} = \overrightarrow{OP_3} - \overrightarrow{OP_1} = \bar{a}_x 2 - \bar{a}_y 4 + \bar{a}_z 4.$
$\overrightarrow{P_1P_2}\cdot\overrightarrow{P_1P_3} = 0. \longrightarrow$ *Right angle at corner* P_1.

b) *Area of triangle* $= \frac{1}{2}\left|\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3}\right| = \frac{1}{2}\left|\overrightarrow{P_1P_2}\right|\left|\overrightarrow{P_1P_3}\right| = 15.3.$

P.2-6 a) $\overrightarrow{P_1P_2} = \bar{a}_x 2 + \bar{a}_y 4 - \bar{a}_z 4,\quad \left|\overrightarrow{P_1P_2}\right| = \sqrt{2^2+4^2+4^2} = 6.$

b) *Perpendicular distance from* P_3 *to the line*
$= \left|\overrightarrow{P_3P_1}\times\bar{a}_{P_1P_2}\right| = \left|(\overrightarrow{OP_1}-\overrightarrow{OP_3})\times\frac{1}{6}\overrightarrow{P_1P_2}\right|$
$= \left|(-\bar{a}_x 5 - \bar{a}_y)\times\frac{1}{6}(\bar{a}_x 2 + \bar{a}_y 4 - \bar{a}_z 4)\right| = \frac{1}{6}\left|\bar{a}_x 4 - \bar{a}_y 20 - \bar{a}_z 18\right| = 4.53.$

P.2-7 Given: $\bar{A} = \bar{a}_x 5 - \bar{a}_y 2 + \bar{a}_z$.

a) Let $\bar{a}_B = \bar{a}_x B_x + \bar{a}_y B_y + \bar{a}_z B_z$,

where $(B_x^2 + B_y^2 + B_z^2)^{1/2} = 1$. (1)

$$\bar{a}_B \parallel \bar{A} \text{ requires } \bar{a}_B \times \bar{A} = 0 = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ B_x & B_y & B_z \\ 5 & -2 & 1 \end{vmatrix},$$

where yields:

$$B_y + 2B_z = 0, \quad (2a)$$

$$-B_x + 5B_z = 0, \quad (2b)$$

$$-2B_x - 5B_y = 0. \quad (2c)$$

Equations (2a), (2b), and (2c) are not all independent.
Solving Eqs. (1) and (2), we obtain

$$B_x = \frac{5}{\sqrt{30}}, \quad B_y = -\frac{2}{\sqrt{30}}, \text{ and } B_z = \frac{1}{\sqrt{30}}$$

$$\therefore \quad \bar{a}_B = \frac{1}{\sqrt{30}}(\bar{a}_x 5 - \bar{a}_y 2 + \bar{a}_z).$$

b) Let $\bar{a}_C = \bar{a}_x C_x + \bar{a}_y C_y + \bar{a}_z C_z$, where $C_z = 0$,

and $C_x^2 + C_y^2 = 1$. (3)

$\bar{a}_C \perp \bar{A}$ requires $\bar{a}_C \cdot \bar{A} = 0$, or

$$5C_x - 2C_y = 0. \quad (4)$$

Solution of Eqs. (3) and (4) yields

$$C_x = \frac{2}{\sqrt{29}}, \text{ and } C_y = \frac{5}{\sqrt{29}}.$$

$$\therefore \quad \bar{a}_C = \frac{1}{\sqrt{29}}(\bar{a}_x 2 + \bar{a}_y 5).$$

P.2-8 Given: $\bar{A} = \bar{A}_1 + \bar{A}_2 = \bar{a}_x 2 - \bar{a}_y 5 + \bar{a}_z 3$,

$$\bar{B} = -\bar{a}_x + \bar{a}_y 4,$$

$$\bar{A}_1 \perp \bar{B} \longrightarrow \bar{A}_1 \cdot \bar{B} = 0,$$

$$\bar{A}_2 \parallel \bar{B} \longrightarrow \bar{A}_2 \times \bar{B} = 0.$$

Solving, we have

$$\bar{A}_1 = \frac{3}{17}(\bar{a}_x 4 + \bar{a}_y + \bar{a}_z 17) \text{ and } \bar{A}_2 = \frac{22}{17}(\bar{a}_x - \bar{a}_y 4).$$

P.2-10 $\overrightarrow{OP_1} = -\bar{a}_x - \bar{a}_z 2,$

$$\overrightarrow{OP_2} = \bar{a}_x (r\cos\phi) + \bar{a}_y (r\sin\phi) + \bar{a}_z z$$
$$= \bar{a}_x \left(-\tfrac{3}{2}\right) + \bar{a}_y \tfrac{\sqrt{3}}{2} + \bar{a}_z .$$
$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = -\bar{a}_x \tfrac{1}{2} + \bar{a}_y \tfrac{\sqrt{3}}{2} + \bar{a}_z 3, \qquad |\overrightarrow{P_1P_2}| = \sqrt{10}.$$

At $P_1(-1, 0, -2)$, $\bar{A}_{P_1} = -\bar{a}_x 2 + \bar{a}_z .$

$$\bar{A}_{P_1} \cdot \bar{a}_{P_1P_2} = \bar{A}_{P_1} \cdot \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \frac{4}{\sqrt{10}} = 1.265$$

P.2-11 a)
$$\left.\begin{aligned} x &= r\cos\phi = 3\cos 240^\circ = -\tfrac{3}{2}, \\ y &= r\sin\phi = 3\sin 240^\circ = -3\sqrt{3}/2, \\ z &= -4 \end{aligned}\right\} \left(-\tfrac{3}{2}, -\tfrac{3\sqrt{3}}{2}, -4\right)$$

b)
$$\left.\begin{aligned} R &= (r^2 + z^2)^{1/2} = (3^2 + 4^2)^{1/2} = 5, \\ \theta &= \tan^{-1}(r/z) = \tan^{-1}\left(\tfrac{3}{-4}\right) = 143.1^\circ, \\ \phi &= 4\pi/3 = 240^\circ. \end{aligned}\right\} (5, 143.1^\circ, 240^\circ)$$

P.2-12 a) $-\sin\phi$, b) $\sin\theta\sin\phi$, c) $\cos\theta$,

d) $-\bar{a}_z \cos\phi$, e) $-\bar{a}_\phi \cos\theta$, f) $-\bar{a}_\phi \cos\theta$.

P.2-13 a) In Cartesian coordinates, $\bar{A} = \bar{a}_x A_x + \bar{a}_y A_y + \bar{a}_z A_z$.

$$A_r = \bar{a}_r \cdot \bar{A} = (\bar{a}_r \cdot \bar{a}_x) A_x + (\bar{a}_r \cdot \bar{a}_y) A_y + (\bar{a}_r \cdot \bar{a}_z) A_z$$
$$= A_x \cos\phi_1 + A_y \sin\phi_1$$

b) In spherical coordinates, $\bar{A} = \bar{a}_R A_R + \bar{a}_\theta A_\theta + \bar{a}_\phi A_\phi$.

$$A_r = \bar{a}_r \cdot \bar{A} = (\bar{a}_r \cdot \bar{a}_R) A_R + (\bar{a}_r \cdot \bar{a}_\theta) A_\theta + (\bar{a}_r \cdot \bar{a}_\phi) A_\phi$$
$$= A_R \sin\theta_1 + A_\theta \cos\theta_1$$
$$= \frac{A_R r_1}{\sqrt{r_1^2 + z_1^2}} + \frac{A_\theta z_1}{\sqrt{r_1^2 + z_1^2}} .$$

P.2-14 a) In Cartesian coordinates, $\bar{E}=\bar{a}_x E_x+\bar{a}_y E_y+\bar{a}_z E_z$.

$$E_\theta=\bar{a}_\theta\cdot\bar{E}=(\bar{a}_\theta\cdot\bar{a}_x)E_x+(\bar{a}_\theta\cdot\bar{a}_y)E_y+(\bar{a}_\theta\cdot\bar{a}_z)E_z$$
$$=E_x\cos\theta_1\cos\phi_1+E_y\cos\theta_1\sin\phi_1-E_z\sin\theta_1.$$

b) In cylindrical coordinates, $\bar{E}=\bar{a}_r E_r+\bar{a}_\phi E_\phi+\bar{a}_z E_z$.

$$E_\theta=\bar{a}_\theta\cdot\bar{E}=(\bar{a}_\theta\cdot\bar{a}_r)E_r+(\bar{a}_\theta\cdot\bar{a}_\phi)E_\phi+(\bar{a}_\theta\cdot\bar{a}_z)E_z$$
$$=E_r\cos\theta_1-E_z\sin\theta_1.$$

P.2-15 a) $\bar{F}_P=\bar{a}_R\dfrac{12}{\sqrt{(-2)^2+(-4)^2+4^2}}=\bar{a}_R\dfrac{12}{6}=\bar{a}_R 2.$

$$(F_P)_y=2\left(\frac{-4}{\sqrt{(-2)^2+(-4)^2+4^2}}\right)=-\frac{4}{3}$$

b) $\bar{a}_F=\frac{1}{6}(-\bar{a}_x 2-\bar{a}_y 4+\bar{a}_z 4)=\frac{1}{3}(-\bar{a}_x-\bar{a}_y 2+\bar{a}_z 2).$

$$\bar{a}_A=\frac{1}{\sqrt{2^2+(-3)^2+(-6)^2}}(\bar{a}_x 2-\bar{a}_y 3-\bar{a}_z 6)=\frac{1}{7}(\bar{a}_x 2-\bar{a}_y 3-\bar{a}_z 6).$$
$$\theta_{FA}=\cos^{-1}(\bar{a}_F\cdot\bar{a}_A)=\cos^{-1}\frac{1}{21}(-2+6-12)=\cos^{-1}\left(\frac{-8}{21}\right)$$
$$=\cos^{-1}(-0.381)=180^\circ-67.6^\circ=112.4^\circ.$$

P.2-16 $\displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_{P_1}^{P_2}(y\,dx+x\,dy).$

a) $x=2y^2,\ dx=4y\,dy;\ \displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_1^2(4y^2dy+2y^2dy)=14.$

b) $x=6y-4,\ dx=6\,dy;\ \displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_1^2[6y\,dy+(6y-4)]dy=14.$

Equal line integrals along two specific paths do not necessarily imply a conservative field. $\bar{E}$ is a conservative field in this case because $\bar{E}=\bar{\nabla}(xy+c)$.

P.2-17 a) $\bar{R}=\bar{a}_x x+\bar{a}_y y+\bar{a}_z z,\quad \dfrac{1}{R}=(x^2+y^2+z^2)^{-1/2}.$

$$\bar{\nabla}\left(\frac{1}{R}\right)=\bar{a}_x\frac{\partial}{\partial x}\left(\frac{1}{R}\right)+\bar{a}_y\frac{\partial}{\partial y}\left(\frac{1}{R}\right)+\bar{a}_z\frac{\partial}{\partial z}\left(\frac{1}{R}\right)$$
$$=-\frac{1}{R^3}(\bar{a}_x x+\bar{a}_y y+\bar{a}_z z)=-\bar{R}/R^3.$$

b) $\bar{R}=\bar{a}_R R,\ \bar{\nabla}\left(\dfrac{1}{R}\right)=\bar{a}_R\dfrac{\partial}{\partial R}\left(\dfrac{1}{R}\right)=-\bar{a}_R\left(\dfrac{1}{R^2}\right)=-\bar{R}/R^3.$

P.2-18 a) $\bar{\nabla} V = \bar{a}_x(2y+z) + \bar{a}_y(2x-z) + \bar{a}_z(x-y)$

$= \bar{a}_x(-2) + \bar{a}_y 4 + \bar{a}_z 3$; Magnitude $= \sqrt{29}$.

b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \bar{a}_x(-2) + \bar{a}_y 3 + \bar{a}_z 6$,

$$\bar{a}_{PQ} = \frac{\overrightarrow{PQ}}{\sqrt{(-2)^2+3^2+6^2}} = \frac{1}{7}(-\bar{a}_x 2 + \bar{a}_y 3 + \bar{a}_z 6).$$

Rate of increase of V from P toward Q $= (\bar{\nabla} V)\cdot \bar{a}_{PQ}$

$$= \frac{1}{7}(4+12+18) = \frac{34}{7}.$$

P.2-19 a) $\frac{\partial \bar{a}_r}{\partial \phi} = \bar{a}_\phi$; $\frac{\partial \bar{a}_\phi}{\partial \phi} = -\bar{a}_r$.

b)
$$\begin{aligned}
\bar{\nabla}\cdot\bar{A} &= \left(\bar{a}_r \frac{\partial}{\partial r} + \bar{a}_\phi \frac{\partial}{r\partial\phi} + \bar{a}_z\frac{\partial}{\partial z}\right)\cdot(\bar{a}_r A_r + \bar{a}_\phi A_\phi + \bar{a}_z A_z) \\
&= \frac{\partial A_r}{\partial r} + \bar{a}_\phi \frac{1}{r}\cdot\frac{\partial}{\partial\phi}(\bar{a}_r A_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z} \\
&= \frac{\partial A_r}{\partial r} + \bar{a}_\phi \frac{1}{r}\cdot\left(\bar{a}_r\frac{\partial A_r}{\partial\phi} + A_r\frac{\partial \bar{a}_r}{\partial\phi}\right) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z} \\
&= \frac{\partial A_r}{\partial r} + \frac{A_r}{r} + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z} \\
&= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}.
\end{aligned}$$

P.2-20 In spherical coordinates,

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R), \quad \text{if } \bar{A} = \bar{a}_R A_R.$$

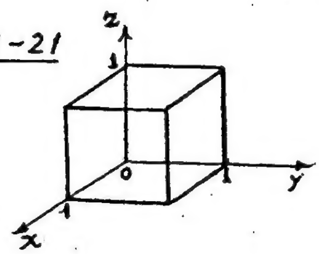
a) $\bar{A} = f_1(\bar{R}) = \bar{a}_R R^n$, $\quad A_R = R^n$.

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^{n+2}) = (n+2)R^{n-1}.$$

b) $\bar{A} = f_2(\bar{R}) = \bar{a}_R \frac{k}{R^2}$, $\quad A_R = kR^{-2}$.

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(k) = 0.$$

P.2-21



$\bar{F} = \bar{a}_x xy + \bar{a}_y yz + \bar{a}_z zx$. To find $\oint \bar{F}\cdot d\bar{s}$:

a) Left face: $y=0$, $d\bar{s} = -\bar{a}_y\, dx\, dz$.

$$\int_0^1\int_0^1 -yz\, dx\, dz = 0. \qquad (1)$$

Right face: $y=1$, $d\bar{s}=\bar{a}_y\,dx\,dz$.

$$\int_0^1\int_0^1 z\,dx\,dz=\frac{1}{2}. \qquad (2)$$

Top face: $z=1$, $d\bar{s}=\bar{a}_z\,dx\,dy$.

$$\int_0^1\int_0^1 dx\,dy=\frac{1}{2}. \qquad (3)$$

Bottom face: $z=0$, $d\bar{s}=-\bar{a}_z\,dx\,dy$, $\int\bar{F}\cdot d\bar{s}=0$. (4)

Front face: $x=1$, $d\bar{s}=\bar{a}_x\,dy\,dz$.

$$\int_0^1\int_0^1 y\,dy\,dz=\frac{1}{2}. \qquad (5)$$

Back face: $x=0$, $d\bar{s}=-\bar{a}_x\,dy\,dz$, $\int\bar{F}\cdot d\bar{s}=0$. (6)

Adding the results in (1), (2), (3), (4), (5), and (6):

$$\oint\bar{F}\cdot d\bar{s}=\frac{3}{2}.$$

b) $\bar{\nabla}\cdot\bar{F}=y+z+x$, $\quad dv=dx\,dy\,dz$.

$$\int\bar{\nabla}\cdot\bar{F}\,dv=\int_0^1\int_0^1\int_0^1(x+y+z)\,dx\,dy\,dz=\frac{3}{2}.$$

<u>P. 2-22</u> $\quad\bar{A}=\bar{a}_r r^2+\bar{a}_z 2z$.

$$\oint_S\bar{A}\cdot d\bar{s}=\left(\int_{\substack{top\\face}}+\int_{\substack{bottom\\face}}+\int_{walls}\right)\bar{A}\cdot d\bar{s}.$$

Top face $(z=4)$: $\bar{A}=\bar{a}_r r^2+\bar{a}_z 8$, $d\bar{s}=\bar{a}_z\,ds$.

$$\int_{\substack{top\\face}}\bar{A}\cdot d\bar{s}=\int_{\substack{top\\face}}8\,ds=8(\pi 5^2)=200\pi.$$

Bottom face $(z=0)$: $\bar{A}=\bar{a}_r r^2$, $d\bar{s}=-\bar{a}_z\,ds$, $\int_{\substack{bottom\\face}}\bar{A}\cdot d\bar{s}=0$.

Walls $(r=5)$: $\bar{A}=\bar{a}_r 25+\bar{a}_z 2z$, $d\bar{s}=\bar{a}_r\,ds$.

$$\int_{Walls}\bar{A}\cdot d\bar{s}=25\int_{Walls}ds=25(2\pi 5\times 4)=1000\pi.$$

$$\therefore\ \oint\bar{A}\cdot d\bar{s}=200\pi+0+1000\pi=1{,}200\pi.$$

$$\bar{\nabla}\cdot\bar{A}=3r+2,\ \int_V\bar{\nabla}\cdot\bar{A}\,dv=\int_0^4\int_0^{2\pi}\int_0^5(3r+2)r\,dr\,d\phi\,dz=1{,}200\pi.$$
$$=\oint\bar{A}\cdot d\bar{s}.$$

P.2-23 $\bar{A} = \bar{a}_z z = \bar{a}_z R\cos\theta$.

a) Over the hemispherical surface: $d\bar{s} = \bar{a}_R R^2 \sin\theta\, d\theta\, d\phi$.

$$\int \bar{A}\cdot d\bar{s} = \int_0^{\pi/2}\int_0^{2\pi} \bar{a}_z (R\cos\theta)\cdot\bar{a}_R R^2 \sin\theta\, d\theta\, d\phi$$

$$= R^3 2\pi \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = \frac{2}{3}\pi R^3.$$

Over the flat base: $z=0$, $\bar{A}=0$, $\int \bar{A}\cdot d\bar{s} = 0$.

$$\therefore \oint \bar{A}\cdot d\bar{s} = \frac{2}{3}\pi R^3.$$

b) $\bar{\nabla}\cdot\bar{A} = \dfrac{\partial A_z}{\partial z} = \dfrac{\partial z}{\partial z} = 1$.

c) $\int \bar{\nabla}\cdot\bar{A}\, dv = 1\times$ (volume of hemispherical region) $= \frac{2}{3}\pi R^3$

$= \oint \bar{A}\cdot d\bar{s} \longrightarrow$ Divergence theorem is proved.

P.2-24 $\bar{D} = \bar{a}_R \dfrac{\cos^2\phi}{R^3}$. $\quad d\bar{s} = \begin{cases} \bar{a}_R R^2 \sin\theta\, d\theta\, d\phi, & \text{at } R=3. \\ -\bar{a}_R R^2 \sin\theta\, d\theta\, d\phi, & \text{at } R=2. \end{cases}$

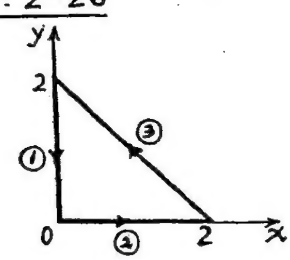
a) $$\oint \bar{D}\cdot d\bar{s} = \int_0^{2\pi}\int_0^{\pi} \left(\frac{1}{3}-\frac{1}{2}\right)\sin\theta\, d\theta\cdot\cos^2\phi\, d\phi$$

$$= -\frac{1}{6}\int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi}\cos^2\phi\, d\phi = -\frac{1}{6}(2)\pi = -\frac{\pi}{3}.$$

b) $\bar{\nabla}\cdot\bar{D} = -\dfrac{\cos^2\phi}{R^4}$, $\quad dv = R^2 \sin\theta\, dR\, d\theta\, d\phi$.

$$\int \bar{\nabla}\cdot\bar{D}\, dv = \int_0^{2\pi}\int_0^{\pi}\int_2^3 \left(-\frac{\cos^2\phi}{R^2}\right)\sin\theta\, dR\, d\theta\, d\phi = -\frac{\pi}{3}.$$

P.2-26



a) $d\bar{\ell} = \bar{a}_x dx + \bar{a}_y dy$,

$\bar{A}\cdot d\bar{\ell} = (2x^2+y^2)dx + (xy-y^2)dy$.

Path ①: $x=0$, $dx=0$, $\int \bar{A}\cdot d\bar{\ell} = -\int_2^0 y^2 dy = 8/3$.

Path ②: $y=0$, $dy=0$, $\int \bar{A}\cdot d\bar{\ell} = \int_0^2 2x^2 dx = 16/3$.

Path ③: $y=2-x$, $dy=-dx$, $\int \bar{A}\cdot d\bar{\ell} = -28/3$.

$$\oint \bar{A}\cdot d\bar{\ell} = \frac{8}{3}+\frac{16}{3}-\frac{28}{3} = -\frac{4}{3}.$$

b) $\bar{\nabla}\times\bar{A} = -\bar{a}_z y$, $d\bar{s} = \bar{a}_z dx\, dy$, $\int(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = -\int_0^2\left[\int_0^{2-x} y\, dy\right]dx = -\frac{4}{3}$.

c) No. $\bar{\nabla}\times\bar{A} \neq 0$.

P.2-27 $\bar{F} = \bar{a}_r 5r\sin\phi + \bar{a}_\phi r^2\cos\phi$.

a) Path AB: $r=1$, $\bar{F} = \bar{a}_r 5\sin\phi + \bar{a}_\phi \cos\phi$; $d\bar{\ell} = \bar{a}_\phi d\phi$.

$$\int_{AB} \bar{F}\cdot d\bar{\ell} = \int_0^{\pi/2} \cos\phi\, d\phi = 1.$$

Path BC: $\phi = \pi/2$, $\bar{F} = \bar{a}_r 5r$; $d\bar{\ell} = \bar{a}_r dr$.

$$\int_{BC} \bar{F}\cdot d\bar{\ell} = \int_1^2 5r\, dr = 15/2.$$

Path CD: $r=2$, $\bar{F} = \bar{a}_r 10\sin\phi + \bar{a}_\phi 4\cos\phi$; $d\bar{\ell} = \bar{a}_\phi 2d\phi$.

$$\int_{CD} \bar{F}\cdot d\bar{\ell} = \int_{\pi/2}^{0} 8\cos\phi\, d\phi = -8.$$

Path DA: $\phi = 0$, $\bar{F} = \bar{a}_\phi r^2$; $d\bar{\ell} = \bar{a}_r dr$.

$$\int_{DA} \bar{F}\cdot d\bar{\ell} = 0.$$

$$\therefore \oint_{ABCDA} \bar{F}\cdot d\bar{\ell} = 1 + \frac{15}{2} - 8 = \frac{1}{2}.$$

b) $\bar{\nabla}\times\bar{F} = \bar{a}_z \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial\phi}\right] = \bar{a}_z(3r-5)\cos\phi.$

c) $d\bar{s} = -\bar{a}_z r\, dr\, d\phi$, $\quad (\bar{\nabla}\times\bar{F})\cdot d\bar{s} = -r(3r-5)dr\cos\phi\, d\phi.$

$$\int(\bar{\nabla}\times\bar{F})\cdot d\bar{s} = -\int_1^2 r(3r-5)dr\int_0^{\pi/2}\cos\phi\, d\phi = \frac{1}{2}.$$

P.2-28 $\bar{A} = \bar{a}_\phi 3\sin(\phi/2)$.

$$\bar{\nabla}\times\bar{A} = \frac{3}{R\sin\theta}\left(\bar{a}_R\cos\theta\sin\frac{\phi}{2} - \bar{a}_\theta\sin\theta\sin\frac{\phi}{2}\right).$$

Assume the hemispherical bowl to be located in the lower half of the xy-plane and its circular rim coincident with the xy-plane. Tracing the rim in a counterclockwise direction, we have

$$d\bar{\ell} = \bar{a}_\phi 4\, d\phi, \quad d\bar{s} = -\bar{a}_R 4^2\sin\theta\, d\theta\, d\phi.$$

$$\oint_C \bar{A}\cdot d\bar{\ell} = \int_0^{2\pi}(\bar{A})_{\substack{R=4\\ \theta=\pi/2}}\cdot(\bar{a}_\phi 4\, d\phi) = \int_0^{2\pi} 12\sin\left(\frac{\phi}{2}\right)d\phi = 48.$$

$$\int_S(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = -12\int_0^{2\pi}\int_{\pi/2}^{\pi}\cos\theta\sin\frac{\phi}{2}\, d\theta\, d\phi = 48.$$

$$= \oint_C \bar{A}\cdot d\bar{\ell}.$$

P.2-30. $\bar{F}=\bar{a}_x(x+3y-c_1z)+\bar{a}_y(c_2x+5z)+\bar{a}_z(2x-c_3y+c_4z)$.

a) $\bar{F}$ is irrotational:

$$\bar{\nabla}\times\bar{F}=\bar{a}_x\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)+\bar{a}_y\left(\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\right)+\bar{a}_z\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)=0.$$

Each component must vanish.

$$\frac{\partial}{\partial y}(2x-c_3y+c_4z)-\frac{\partial}{\partial z}(c_2x+5z)=0 \longrightarrow c_3=-5.$$

$$\frac{\partial}{\partial z}(x+3y-c_1z)-\frac{\partial}{\partial x}(2x-c_3y+c_4z)=0 \longrightarrow c_1=-2,$$

$$\frac{\partial}{\partial x}(c_2x+5z)-\frac{\partial}{\partial y}(x+3y-c_1z)=0 \longrightarrow c_2=3.$$

b) F is also solenoidal:

$$\bar{\nabla}\cdot\bar{F}=\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}=0.$$

$$\frac{\partial}{\partial x}(x+3y-c_1z)+\frac{\partial}{\partial y}(c_2x+5z)+\frac{\partial}{\partial z}(2x-c_3y+c_4z)=0.$$

$$\longrightarrow c_4=-1.$$

Chapter 3

Static Electric Fields

P. 3-1 a) Max. voltage V_{max} will make $d_1 = h/2$ at $z = w$.

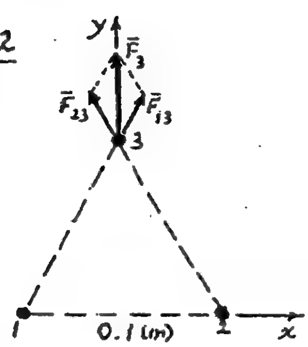
$$\frac{h}{2} = \frac{e}{2m}\left(\frac{V_{max}}{h}\right)\left(\frac{w}{u_0}\right)^2 \longrightarrow V_{max} = \frac{m}{e}\left(\frac{u_0 h}{w}\right)^2.$$

b) At the screen, $(d_0)_{max} = D/2$. L must be $\leq L_{max}$, where

$$L_{max} = \frac{1}{2}\left(w + \frac{m u_0^2 D h}{e w V_{max}}\right).$$

c) Double V_{max} by doubling u_0^2, or doubling the anode accelerating voltage.

P. 3-2



$$\bar{F}_{13} = \frac{(2\times10^{-6})^2}{4\pi\epsilon_0\,(0.1)^2}\,(\bar{a}_x 0.5 + \bar{a}_y 0.866)$$

$$= 3.6\,(\bar{a}_x 0.5 + \bar{a}_y 0.866)\ \ (N).$$

$$\bar{F}_{23} = 3.6\,(-\bar{a}_x 0.5 + \bar{a}_y 0.866)\ \ (N).$$

$$\bar{F}_3 = \bar{F}_{13} + \bar{F}_{23} = \bar{a}_y\, 0.624\ \ (N).$$

Similarly for $\bar{F}_1$ and $\bar{F}_2$. All are repulsive forces in the direction away from the center of the triangle.

P. 3-3 $\overline{Q_1P} = -\bar{a}_y 3 + \bar{a}_z 4$, $\quad \overline{Q_2P} = \bar{a}_y 4 - \bar{a}_z 3$.

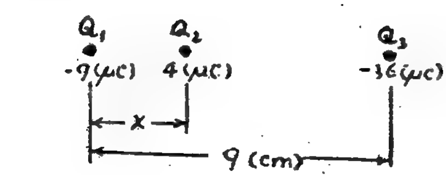
At P: $\bar{E}_1 = \dfrac{Q_1}{4\pi\epsilon_0 (5)^3}(-\bar{a}_y 3 + \bar{a}_z 4)$,

$$\bar{E}_2 = \frac{Q_2}{4\pi\epsilon_0 (5)^3}(\bar{a}_y 4 - \bar{a}_z 3).$$

a) No y-component: $-3Q_1 + 4Q_2 = 0 \longrightarrow \dfrac{Q_1}{Q_2} = \dfrac{4}{3}$.

b) No z-component: $4Q_1 - 3Q_2 = 0 \longrightarrow \dfrac{Q_1}{Q_2} = \dfrac{3}{4}$.

P. 3-4



For zero force on Q_1:

$$\frac{Q_1 Q_2}{4\pi\epsilon_0 x^2} + \frac{Q_1 Q_3}{4\pi\epsilon_0 9^2} = 0.$$

$$x = 9\sqrt{-\frac{Q_2}{Q_3}} = 9\sqrt{\frac{4}{36}} = 3 \text{ (cm)}.$$

With $x = 3$ (cm), it can be proved that the net forces on Q_2 and Q_3 are also zero.

P. 3-5 From Eq. (3-42a), $\rho_s = \frac{\text{Total charge}}{\text{Disk area}} = \frac{Q}{\pi b^2}$.

$$\bar{E} = \bar{a}_z \frac{\rho_s}{2\epsilon_0}\left[1 - \left(1 + \frac{b^2}{z^2}\right)^{-1/2}\right]$$

$$= \bar{a}_z \frac{\rho_s}{2\epsilon_0}\left[1 - \left(1 - \frac{b^2}{2z^2} + \frac{3}{8}\frac{b^4}{z^4} - \cdots\right)\right]$$

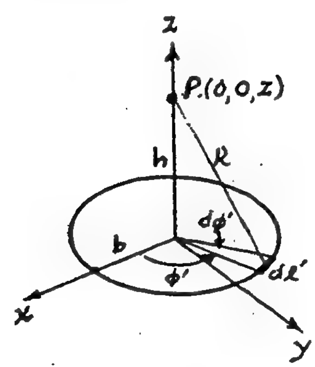
$$= \bar{a}_z \frac{Q}{2\pi\epsilon_0 b^2}\left[\frac{1}{2}\left(\frac{b}{z}\right)^2 - \frac{3}{8}\left(\frac{b}{z}\right)^4 + \cdots\right]$$

$$= \bar{a}_z\left[\frac{Q}{4\pi\epsilon_0 z^2}\left(1 - \frac{3}{4}\frac{b^2}{z^2} + \cdots\right)\right],$$

where the first term is the point-charge term and the rest represent the error. Considering only the first error term:

$$\frac{3}{4}\cdot\frac{b^2}{z^2} \le 0.01. \longrightarrow z \ge \sqrt{75}\, b, \text{ or } 8.66\, b.$$

P. 3-6 At an arbitrary $P(0,0,z)$ on the axis:



$$dV_P = \frac{\rho_\ell b\, d\phi'}{4\pi\epsilon_0 (z^2+b^2)^{1/2}}.$$

$$V_P = \frac{\rho_\ell b}{4\pi\epsilon_0 (z^2+b^2)^{1/2}}\int_0^{2\pi} d\phi' = \frac{\rho_\ell b}{2\epsilon_0 (z^2+b^2)^{1/2}}.$$

$$\bar{E}_P = -\bar{\nabla} V_P = -\bar{a}_z \frac{dV_P}{dz} = \bar{a}_z \frac{\rho_\ell b}{2\epsilon_0 (z^2+b^2)^{3/2}}.$$

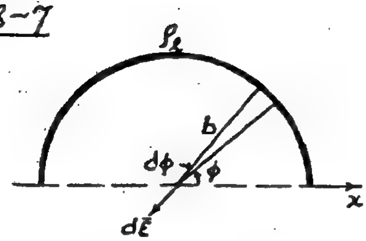
a) At point $(0,0,h)$, $\bar{E} = \bar{a}_z \frac{\rho_\ell b}{2\epsilon_0 (h^2+b^2)^{3/2}}$.

b) To find the location of max. $|\bar{E}_P|$, set

$$\frac{\partial}{\partial z}|\bar{E}_P| = 0 \longrightarrow z = \frac{b}{\sqrt{2}}. \quad \text{Max}|\bar{E}_P| = \frac{\rho_\ell}{3.67\epsilon_0 b^2}.$$

Similar situation when P is below the loop.

P.3-7



$$dE_y = -\frac{\rho_\ell (b\,d\phi)}{4\pi\epsilon_0 b^2}\sin\phi,$$

$$\bar{E} = \bar{a}_y E_y = -\bar{a}_y \frac{\rho_\ell}{4\pi\epsilon_0 b}\int_0^{\pi}\sin\phi\,d\phi$$

$$= -\bar{a}_y \frac{\rho_\ell}{2\pi\epsilon_0 b}.$$

P.3-8 Spherical symmetry: $\bar{E} = \bar{a}_R E_R$. Apply Gauss's law.

1) $0 \leq R \leq b$. $4\pi R^2 E_{R1} = \frac{\rho_0}{\epsilon_0}\int_0^R \left(1 - \frac{R^2}{b^2}\right) 4\pi R^2 dR = \frac{4\pi\rho_0}{\epsilon_0}\left(\frac{R^3}{3} - \frac{R^5}{5b^2}\right),$

$$E_{R1} = \frac{\rho_0}{\epsilon_0} R\left(\frac{1}{3} - \frac{R^2}{5b^2}\right).$$

2) $b \geqq R < R_i$. $4\pi R^2 = \frac{\rho_0}{\epsilon_0}\int_0^b \left(1 - \frac{R^2}{b^2}\right) 4\pi R^2 dR = \frac{8\pi\rho_0}{15\epsilon_0} b^3,$

$$E_{R2} = \frac{2\rho_0 b^3}{15\epsilon_0 R^2}.$$

3) $R_i < R < R_o$. $E_{R3} = 0.$

4) $R > R_o$. $E_{R4} = \frac{2\rho_0 b^3}{15\epsilon_0 R^2}.$

P.3-9 Cylindrical symmetry: $\bar{E} = \bar{a}_r E_r$. Apply Gauss's law.

a) $E_r = 0$, for $r < a$.

$E_r = \frac{a\rho_{sa}}{\epsilon_0 r}$, for $a < r < b$.

$E_r = \frac{a\rho_{sa} + b\rho_{sb}}{\epsilon_0 r}$, for $r > b$.

b) $\frac{b}{a} = -\frac{\rho_{sa}}{\rho_{sb}}.$

P.3-10 $W_e = -q\int \bar{E}\cdot d\bar{\ell} = -q\int (y\,dx + x\,dy).$

a) Along the parabola $y = 2x^2$, $dy = 4x\,dx$.

$$W_e = -(5\times10^{-6})\int_1^{-2}(2x^2 + 4x^2)\,dx = 9\times10^{-5}\,(J) = 90\,(\mu J).$$

b) Along the straight line $\frac{y-2}{x-1} = \frac{8-2}{-2-1} = -2$, $y = -2x + 4$, $dy = -2dx$.

$$W_e = -(5\times10^{-6})\int_1^{-2}[(-2x+4)\,dx - 2x\,dx] = 90\,(\mu J).$$

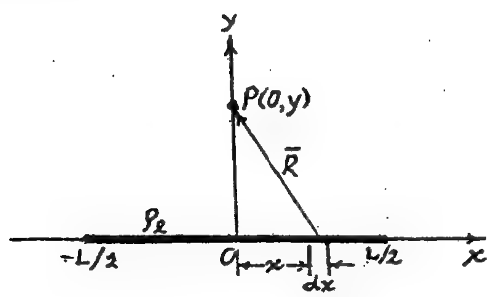
P.3-11 $\bar{E} = \bar{a}_x y - \bar{a}_y x$, $\quad \bar{E}\cdot d\bar{l} = y\,dx - x\,dy$.

a) $W_e = -q\int_1^{-2}(2x^2 - 4x^2)\,dx = -30\,(\mu J)$.

b) $W_e = -q\int_1^{-2}[(-2x+4)+2x]\,dx = -60\,(\mu J)$.

The given $\bar{E}$ field is nonconservative.

P.3-12



a) $V = 2\int_0^{L/2}\frac{\rho_\ell\,dx}{4\pi\epsilon_0 R}$

$= \frac{\rho_\ell}{2\pi\epsilon_0}\int_0^{L/2}\frac{dx}{\sqrt{x^2+y^2}}$

$= \frac{\rho_\ell}{2\pi\epsilon_0}\left\{\ln\left[\sqrt{\left(\frac{L}{2}\right)^2+y^2}+\frac{L}{2}\right] - \ln y\right\}$.

b) From Coulomb's law:

$$\bar{E} = \bar{a}_y E_y = 2\int_0^{L/2}\bar{a}_y\frac{\rho_\ell\,y\,dx}{4\pi\epsilon_0 R^3} = \bar{a}_y\frac{\rho_\ell}{2\pi\epsilon_0 y}\left[\frac{L/2}{\sqrt{(L/2)^2+y^2}}\right].$$

c) $\bar{E} = -\bar{\nabla}V$ gives the same answer as in b).

P.3-13 a) $\rho_{ps} = \bar{P}\cdot\bar{a}_n = P_0\frac{L}{2}$ on all six faces of the cube.

$\rho_{pv} = -\bar{\nabla}\cdot\bar{P} = -3P_0$.

b) $Q_s = (6L^2)\rho_{ps} = 3P_0L^2$; $\quad Q_v = (L^3)\rho_{pv} = -3P_0L^3$.

Total bound charge $= Q_s + Q_v = 0$.

P.3-14 $\bar{P} = \bar{a}_x P_0$.

a) $\rho_{ps} = \bar{P}\cdot\bar{a}_R = P_0\sin\theta\cos\phi$.

$\rho_{pv} = -\bar{\nabla}\cdot\bar{P} = 0$.

b) $Q_s = \int_0^{\pi}\int_0^{2\pi} P_0\, b^2\sin^2\theta\cos\phi\,d\phi\,d\theta$

$= 0$.

P.3-15 $\bar{P} = P_0(\bar{a}_x 3x + \bar{a}_y 4y)$.

a) $\rho_{pv} = -\bar{\nabla}\cdot\bar{P} = -7P_0$.

Total volume charge $Q_v = -7P_0\pi(r_o^2 - r_i^2)$ per unit length.

Outer surface: $r = r_o$, $\bar{a}_n = \bar{a}_r$. $\rho_{ps_o} = \bar{P}\cdot\bar{a}_r = P_0(\bar{a}_x 3r_o\cos\phi + \bar{a}_y 4r_o\sin\phi)\cdot\bar{a}_r$

$$= P_0 r_o(3\cos^2\phi + 4\sin^2\phi)$$

$$= P_0 r_o(3 + \sin^2\phi).$$

Inner surface: $r = r_i$, $\bar{a}_n = -\bar{a}_r$. $\rho_{ps_i} = -P_0 r_i(3 + \sin^2\phi)$.

b) Total $Q_{so} = \int_0^{2\pi}\rho_{ps_o} r_o\, d\phi = P_0 r_o^2\int_0^{2\pi}(3 + \sin^2\phi)\,d\phi = 7\pi P_0 r_o^2$, per unit length.

Total $Q_{si} = -7\pi P_0 r_i^2$, per unit length.

Total bound charge: $Q_v + Q_{so} + Q_{si} = 0$.

P.3-16 Spherical symmetry: Apply Gauss's law. $\bar{E} = \bar{a}_R E_R$, $\bar{D} = \bar{a}_R D_R$.

(1) $R > R_o$.

$$E_{R1} = \frac{Q}{4\pi\epsilon_0 R^2}, \qquad V_1 = \frac{Q}{4\pi\epsilon_0 R}.$$

$$D_{R1} = \epsilon_0 E_{R1} = \frac{Q}{4\pi R^2}, \qquad P_{R1} = 0.$$

(2) $R_i < R < R_o$.

$$E_{R2} = \frac{Q}{4\pi\epsilon_0\epsilon_r R^2}, \qquad D_{R2} = \frac{Q}{4\pi R^2}.$$

$$P_{R2} = D_{R2} - \epsilon_0 E_{R2} = \left(1 - \frac{1}{\epsilon_r}\right)\frac{Q}{4\pi R^2}.$$

$$V_2 = -\int_\infty^{R_o} E_{R1}\,dR - \int_{R_o}^{R} E_{R2}\,dR = \frac{Q}{4\pi\epsilon_0}\left[\left(1 - \frac{1}{\epsilon_r}\right)\frac{1}{R_o} - \frac{1}{\epsilon_r R}\right].$$

(3) $R < R_i$

$$E_{R3} = \frac{Q}{4\pi\epsilon_0 R^2}, \quad D_{R3} = \frac{Q}{4\pi R^2}, \quad P_{R3} = 0.$$

$$V_3 = V_2\Big|_{R=R_i} - \int_{R_i}^{R} E_{R3}\,dR$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\left(1 - \frac{1}{\epsilon_r}\right)\frac{1}{R_o} - \left(1 - \frac{1}{\epsilon_r}\right)\frac{1}{R_i} + \frac{1}{R}\right].$$

P.3-17 Use subscript a for air, p for plexiglass, and b for breakdown.

a) $V_b = E_{ba} d_a = (3\times10^6)\times(50\times10^{-3}) = 150\times10^3 (V) = 150\ (kV)$.

b) $V_b = E_{bp} d_p = 20\times50 = 1{,}000\ (kV)$.

c) $V_b = E_a d_a + E_p d_p = E_a(50-d_p) + E_p d_p$

Now $D_a = D_p \longrightarrow \epsilon_0 E_a = \epsilon_0\epsilon_{rp}E_p \longrightarrow E_a = \epsilon_{rp}E_p > E_p$.

$E_{ba} < E_{bp} \longrightarrow$ Breakdown occurs in air region first.

$\therefore\ V_b = E_{ba}(50-10) + \frac{E_{ba}}{3}\times10 = 3(40-\frac{1}{3}\times10) = 130\ (kV)$.

P.3-18 At the $z=0$ plane: $\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z 5$.

$\bar{E}_{1t}(z=0) = \bar{E}_{2t}(z=0) = \bar{a}_x 2y - \bar{a}_y 3x$.

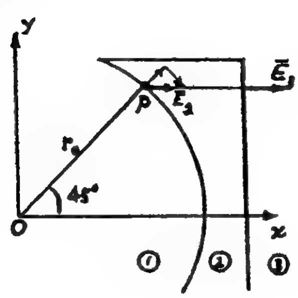
$\bar{D}_{1n}(z=0) = \bar{D}_{2n}(z=0) \longrightarrow 2\bar{E}_{1n}(z=0) = 3\bar{E}_{2n}(z=0)$,

$\longrightarrow \bar{E}_{2n}(z=0) = \frac{2}{3}(\bar{a}_z 5) = \bar{a}_z\frac{10}{3}$.

$\therefore\ \bar{E}_2(z=0) = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z\frac{10}{3}$.

$\bar{D}_2(z=0) = (\bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z\frac{10}{3})3\epsilon_0$

P.3-19



Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$.

Boundary condition: $\bar{a}_n\times\bar{E}_1 = \bar{a}_n\times\bar{E}_2$.

$\longrightarrow E_{2\phi} = -3$.

For $\bar{E}_3$, and hence $\bar{E}_2$, to be parallel to the x-axis, $E_{2\phi} = -E_{2r}$.

$\longrightarrow E_{2r} = 3$.

Boundary condition: $\bar{a}_n\cdot\bar{D}_1 = \bar{a}_n\cdot\bar{D}_2$.

$\longrightarrow \epsilon_1 E_{r1} = \epsilon_2 E_{r2} \longrightarrow \epsilon_0 5 = \epsilon_0\epsilon_{r2}3$.

$\longrightarrow \epsilon_{r2} = \frac{5}{3} = 1.667$.

P.3-20 $\epsilon = \frac{\epsilon_2-\epsilon_1}{d}y+\epsilon_1$.

Assume Q on plate at $y=d$. $\bar{E}=-\bar{a}_y\frac{\rho_s}{\epsilon} = \frac{Q}{S(\frac{\epsilon_2-\epsilon_1}{d}y+\epsilon_1)}$.

$$V=-\int_{y=0}^{y=d}\bar{E}\cdot d\bar{\ell} = \frac{Qd\ln(\epsilon_2/\epsilon_1)}{S(\epsilon_2-\epsilon_1)}.$$

$$C=\frac{Q}{V}=\frac{S(\epsilon_2-\epsilon_1)}{d\ln(\epsilon_2/\epsilon_1)}.$$

P.3-21 Let ρ_ℓ be the lineal charge density on the inner conductor. $\bar{E}=\bar{a}_r\frac{\rho_\ell}{2\pi\epsilon r}$.

$$V_0=-\int_b^a\bar{E}\cdot d\bar{r}=\frac{\rho_\ell}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)\longrightarrow \rho_\ell=\frac{2\pi\epsilon V_0}{\ln(b/a)}.$$

a) $\bar{E}(a)=\bar{a}_r\frac{V_0}{a\ln(b/a)}$.

b) For a fixed b, the function to be minimized is: $(x=b/a)$:
$f(x)=\frac{V_0 x}{b\cdot\ln x}$. Setting $\frac{df(x)}{dx}=0$ yields $\ln x=1$,
or $x=\frac{b}{a}=e=2.718$.

c) min. $E(a)=eV_0/b$.

d) $C'=\frac{\rho_\ell}{V_0}=\frac{2\pi\epsilon}{\ln(b/a)}=2\pi\epsilon$. (F/m).

P.3-22 $\bar{D}=\bar{a}_r\frac{\rho_\ell}{2\pi r}$. $\bar{E}_1=\bar{a}_r\frac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r1}r}$, $r_i<r<b$;

$\bar{E}_2=\bar{a}_r\frac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r2}r}$, $b<r<r_o$.

$$V=-\int_{r_o}^{r_i}\bar{E}\cdot d\bar{r}=\frac{\rho_\ell}{2\pi\epsilon_0}\left[\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_i}\right)+\frac{1}{\epsilon_{r2}}\ln\left(\frac{r_o}{b}\right)\right],$$

$$C'=\frac{\rho_\ell}{V}=\frac{2\pi\epsilon_0}{\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_i}\right)+\frac{1}{\epsilon_{r2}}\ln\left(\frac{r_o}{b}\right)}\quad\text{(F/m)}.$$

P.3-23 Assume charges $+Q$ and $-Q$ on the inner and outer conductors, respectively. $\bar{E}=\bar{a}_R E_R=\bar{a}_R\frac{Q}{4\pi\epsilon_0 R^2}$.

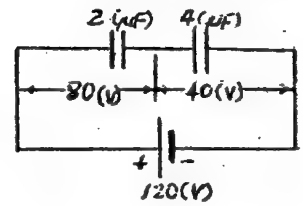
$$V=-\int_{R_o}^{R_i}\bar{E}\cdot\bar{a}_R\,dR=\frac{Q}{4\pi\epsilon}\left(\frac{1}{R_i}-\frac{1}{R_o}\right).$$

$$C=\frac{Q}{V}=\frac{4\pi\epsilon}{(1/R_i-1/R_o)}.$$

P.3-24 Total capacitance across battery terminals,

$$C_T = \frac{4}{3}\,(\mu F).$$

Total stored eletric energy $W_T = \frac{1}{2}C_T(120)^2 = 9.6\,(mW)$.



W_e in 2-(μF) capacitor $= \frac{1}{2}(2\times10^{-6})\times 80^2 = 6.4\,(mW)$.

W_e in 1-(μF) capacitor $= \frac{1}{2}(10^{-6})\times 40^2 = 0.8\,(mW)$.

W_e in 3-(μF) capacitor $= \frac{1}{2}(3\times10^{-6})\times 40^2 = 2.4\,(mW)$.

P.3-25

$$\bar{E} = \bar{a}_r 6r\sin\phi + \bar{a}_\phi 3r\cos\phi,$$

$$d\bar{\ell} = \bar{a}_r\,dr + \bar{a}_\phi r\,d\phi + \bar{a}_z\,dz,$$

$$W_e = -Q\int_{P_1}^{P_2}\bar{E}\cdot d\bar{\ell} = -(5\times10^{-10})\left[6\sin\phi\int_2^4 r\,dr + 3r^2\int_{\pi/3}^{-\pi/2}\cos\phi\,d\phi\right].$$

a) First $r=2$, ϕ from $\pi/3$ to $-\pi/2$; then $\phi=-\pi/2$, r from 2 to 4:

$$W_e = -(5\times10^{-10})\left[3(2)^2\sin\phi\Big|_{\pi/3}^{-\pi/2} + 6\sin\left(-\frac{\pi}{2}\right)\frac{r^2}{2}\Big|_2^4\right]$$

$$= -(5\times10^{-10})\left[-18-36\right] = 27\times10^{-9}\,(J) = 27\,(nJ).$$

b) First $\phi=\pi/3$, r from 2 to 4; then $r=4$, ϕ from $\pi/3$ to $-\pi/2$:

$$W_e = -(5\times10^{-10})\left[6\sin\left(\frac{\pi}{3}\right)\frac{r^2}{2}\Big|_2^4 + 3(4)^2\sin\phi\Big|_{\pi/3}^{-\pi/2}\right]$$

$$= -(5\times10^{-10})\left[18-72\right] = 27\times10^{-9}\,(J) = 27\,(nJ).$$

—— Same as W_e in part a). $\nabla\times\bar{E}=0 \rightarrow \bar{E}$ is conservative.

P.3-26 Assume the inner and outer radii to be a and $a+r$ respectively. Substituting Eq.(3-89) in Eq.(3-117) and using Eq.(3-115), we have

$$F_a = -\frac{\partial}{\partial r}\left(\frac{Q}{2}\cdot\frac{Q}{2\pi\epsilon L}\ln\frac{a+r}{a}\right)$$

$$= -\frac{Q^2}{4\pi\epsilon L(a+r)} = -\frac{Q^2}{4\pi\epsilon L b},$$ in the direction of decreasing r (attraction).

P.3-27 Switch open: Charges on the plates are constant.

$$Q = CV_0 \,, \quad W_e = \frac{Q^2}{2C}.$$

$$C = \frac{w}{d}\left[\epsilon x + \epsilon_0 (L-x)\right].$$

$$\bar{F}_e = -\bar{\nabla} W_e = -\bar{a}_x \frac{Q^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{C}\right)$$

$$= \bar{a}_x \frac{Q^2 d}{2w}\frac{\epsilon-\epsilon_0}{[\epsilon x + \epsilon_0(L-x)]^2} = \bar{a}_x \frac{V_0^2 w}{2d}(\epsilon-\epsilon_0).$$

P.3-28 Use subscripts d and a to denote dielectric and air regions respectively. $\bar{\nabla}^2 V = 0$ in both regions.

$$V_d = c_1 y + c_2, \quad \bar{E}_d = -\bar{a}_y c_1, \quad \bar{D}_d = -\bar{a}_y \epsilon_0 \epsilon_r c_1.$$

$$V_a = c_3 y + c_4, \quad \bar{E}_a = -\bar{a}_y c_3, \quad \bar{D}_a = -\bar{a}_y \epsilon_0 c_3.$$

B.C.: At $y=0$, $V_d = 0$; at $y=d$, $V_a = V_0$;

at $y = 0.8d$: $V_d = V_a$, $\bar{D}_d = \bar{D}_a$.

Solving: $c_1 = \dfrac{V_0}{(0.8+0.2\epsilon_r)d}$, $c_2 = 0$, $c_3 = \dfrac{\epsilon_r V_0}{(0.8+0.2\epsilon_r)d}$, $c_4 = \dfrac{(1-\epsilon_r)V_0}{1+0.25\epsilon_r}$.

a) $V_d = \dfrac{5yV_0}{(4+\epsilon_r)d}$, $\quad \bar{E}_d = -\bar{a}_y \dfrac{5V_0}{(4+\epsilon_r)d}$.

b) $V_a = \dfrac{5\epsilon_r y - 4(\epsilon_r - 1)d}{(4+\epsilon_r)d} V_0$, $\quad \bar{E}_a = -\bar{a}_y \dfrac{5\epsilon_r V_0}{(4+\epsilon_r)d}$.

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = \dfrac{5\epsilon_0\epsilon_r V_0}{(4+\epsilon_r)d}$.

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\dfrac{5\epsilon_0\epsilon_r V_0}{(4+\epsilon_r)d}$.

P.3-29 Poisson's eq. $\bar{\nabla}^2 V = -\dfrac{A}{\epsilon r} \longrightarrow \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial V}{\partial r}\right) = -\dfrac{A}{\epsilon r}$.

Solution: $V = -\dfrac{A}{\epsilon} r + c_1 \ln r + c_2$.

B.C.: $\begin{cases} \text{At } r=a, \ V_0 = -\dfrac{A}{\epsilon} a + c_1 \ln a + c_2. \\ \text{At } r=b, \ 0 = -\dfrac{A}{\epsilon} b + c_1 \ln b + c_2. \end{cases}$ $\quad c_1 = \dfrac{\frac{A}{\epsilon}(b-a) - V_0}{\ln(b/a)}$, $\quad c_2 = \dfrac{V_0 \ln b + \frac{A}{\epsilon}(a \ln b - b \ln a)}{\ln(b/a)}$.

P.3-30 $$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)=0.$$

Solution: $V = c_1 \ln r + c_2$.

Boundary conditions:

$$\text{At } r=a,\ V=V_0 = c_1 \ln a + c_2.$$

$$\text{At } r=0,\ V=0 = c_1 \ln b + c_2.$$

$$c_1 = -\frac{V_0}{\ln(b/a)},\quad c_2 = \frac{V_0 \ln b}{\ln(b/a)}.$$

$$V = V_0\frac{\ln(b/r)}{\ln(b/a)},\quad \bar{E} = -\bar{\nabla}V = \bar{a}_r\frac{V_0}{r\ln(b/a)}.$$

Surface densities: At $r=a$, $\rho_{sa} = \epsilon_0 E_r = \dfrac{\epsilon_0 V_0}{a\ln(b/a)}$.

At $r=b$, $\rho_{sb} = -\epsilon_0 E_r = -\dfrac{\epsilon_0 V_0}{b\ln(b/a)}$.

Capacitance per unit length $$C' = \frac{Q}{V_{ab}} = \frac{2\pi a \rho_{sa}}{V_0} = \frac{2\pi\epsilon_0}{\ln(b/a)} \quad (\text{C/m}).$$

P.3-31 V and $\bar{E}$ depend only on θ. $\longrightarrow$ Eq.(3-129): $\dfrac{d}{d\theta}\left(\sin\theta\dfrac{dV}{d\theta}\right)=0$.

a) Solution: $\dfrac{dV}{d\theta} = \dfrac{C_1}{\sin\theta} \longrightarrow V(\theta) = C_1\ln\left(\tan\frac{\theta}{2}\right)+C_2$.

B.C. ① $V(\alpha) = V_0 = C_1\ln\left(\tan\frac{\theta}{2}\right)+C_2$.

② $V\left(\frac{\pi}{2}\right) = 0 = C_1\ln\left(\tan\frac{\pi}{4}\right)+C_2 \longrightarrow C_2 = 0$.

$\longrightarrow C_1 = \dfrac{V_0}{\ln[\tan(\alpha/2)]} \longrightarrow V(\theta) = \dfrac{V_0\ln[\tan(\theta/2)]}{\ln[\tan(\alpha/2)]}$.

b) $$\bar{E} = -\bar{a}_\theta\frac{dV}{R\,d\theta} = -\bar{a}_\theta\frac{V_0}{R\ln[\tan(\alpha/2)]\sin\theta}.$$

c) On the cone: $\theta=\alpha$, $\rho_s = \epsilon_0 E(\alpha) = -\dfrac{\epsilon_0 V_0}{R\ln[\tan(\alpha/2)]\sin\alpha}$.

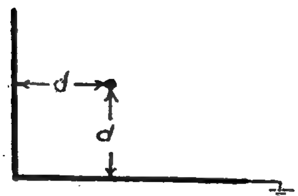
On the grounded plane: $\theta=\pi/2$, $\rho_s = -\epsilon_0 E\left(\frac{\pi}{2}\right) = \dfrac{\epsilon_0 V_0}{R\ln[\tan(\alpha/2)]}$.

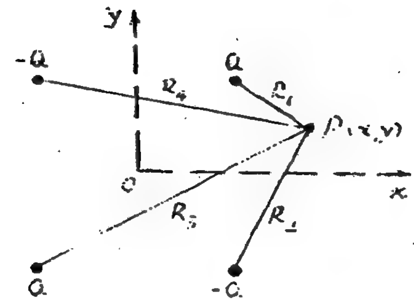
P.3-32 Consider the conditions in the xy-plane ($z=0$).

a) $V_P = \dfrac{Q}{4\pi\epsilon}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}+\dfrac{1}{R_3}-\dfrac{1}{R_4}\right)$, where

$R_1 = [(x-d)^2+(y-d)^2]^{1/2}$, $\quad R_2 = [(x-d)^2+(y+d)^2]^{1/2}$,

$R_3 = [(x+d)^2+(y+d)^2]^{1/2}$, $\quad R_4 = [(x+d)^2+(y-d)^2]^{1/2}$.





$$\bar{E}_p = -\bar{\nabla} V_p = -\bar{a}_x \frac{\partial V_p}{\partial x} - \bar{a}_y \frac{\partial V_p}{\partial y}$$

$$= \bar{a}_x \frac{Q}{4\pi\epsilon}\left[-\frac{x-d}{R_1^3} + \frac{x-d}{R_2^3} - \frac{x+d}{R_3^3} + \frac{x+d}{R_4^3}\right]$$

$$+ \bar{a}_y \frac{Q}{4\pi\epsilon}\left[-\frac{y-d}{R_1^3} + \frac{y+d}{R_2^3} - \frac{y+d}{R_3^3} + \frac{y-d}{R_4^3}\right].$$

E_p will have a z-component if the point P does not lie in the xy-plane.

b) On the conducting half-planes, $\rho_s = D_n = \epsilon E_n$.
Along the x-axis, $y=0$: $R_1 = [(x-d)^2+d^2]^{1/2} = R_2$,
and $R_3 = [(x+d)^2+d^2]^{1/2} = R_4$:

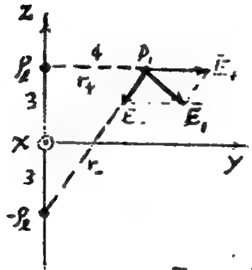
$$E_x = 0, \quad E_y = \frac{Q}{2\pi\epsilon}\left[\frac{d}{R_1^3} - \frac{d}{R_3^3}\right].$$

$$\therefore \quad \rho_s(y=0) = \frac{Qd}{2\pi}\left\{\frac{1}{[(x-d)^2+d^2]^{3/2}} - \frac{1}{[(x+d)^2+d^2]^{3/2}}\right\}$$

$$= \begin{cases} 0, & \text{at } x=0. \\ \text{max.}, & \text{at } x=d. \end{cases}$$

Similarly for $\rho_s(x=0)$ on the vertical conducting plane by changing x to y.

P.3-34



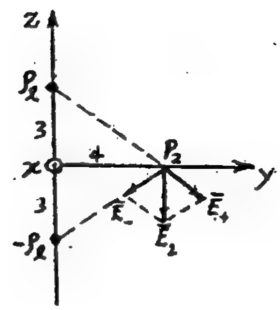
Assume ρ_ℓ (50 nC/m) to be at $y=0$ and $z=3$ (m)

a) Vector from ρ_ℓ to $P_1(0,4,3)$: $\bar{r}_+ = \bar{a}_y 4$.
Vector from $-\rho_\ell$ to P_1: $\bar{r}_- = \bar{a}_y 4 + \bar{a}_z 6$.

$$\bar{E}_+ = \frac{\rho_\ell}{2\pi\epsilon_0}\frac{\bar{r}_+}{r_+^2} = \frac{\rho_\ell}{2\pi\epsilon_0}\frac{\bar{a}_y}{4} = 9\times10^5(\bar{a}_y 0.25)$$

$$\bar{E}_- = \frac{-\rho_\ell}{2\pi\epsilon_0}\frac{\bar{r}_-}{r_-^2} = -9\times10^5(\bar{a}_y 0.077 + \bar{a}_z 0.115).$$

$$\bar{E}_1 = \bar{E}_+ + \bar{E}_- = 9\times10^5(\bar{a}_y 0.173 - \bar{a}_z 0.115) \text{ (V/m) at } P_1.$$



b) At P_2 (0,4,0) on the xy-plane (the ground):

Vector from ρ_ℓ to P_2 is $\bar{r}_+ = \bar{a}_y 4 - \bar{a}_z 3$,

Vector from $-\rho_\ell$ to P_2 is $\bar{r}_- = \bar{a}_y 4 + \bar{a}_z 3$.

$$\bar{E}_+ = \frac{\rho_\ell}{2\pi\epsilon_0}\,\frac{\bar{a}_y 4 - \bar{a}_z 3}{4^2+3^2}, \quad \bar{E}_- = \frac{-\rho_\ell}{2\pi\epsilon_0}\,\frac{\bar{a}_y 4 + \bar{a}_z 3}{4^2+3^2}.$$

$$\bar{E}_2 = \bar{E}_+ + \bar{E}_- = \frac{\rho_\ell}{2\pi\epsilon_0}\left(\frac{-\bar{a}_z 6}{4^2+3^2}\right)$$

$$= 9\times 10^5(-\bar{a}_z 0.24) = -\bar{a}_z 2.16\times 10^5 \text{ (V/m)}.$$

$$\rho_{s2} = \epsilon_0 E_{2z} = \frac{\rho_\ell}{2\pi}(-0.24) = \frac{50\times 10^{-6}}{2\pi}(0.24) = -1.91\times 10^{-6} \text{ (C/m}^2\text{)}$$

$$= -1.91 \;(\mu\text{C/m}^2).$$

<u>P.3-35</u> Given $D = 2$ (cm), $a = 0.3$ (cm).

a) From Eq. (3-163),

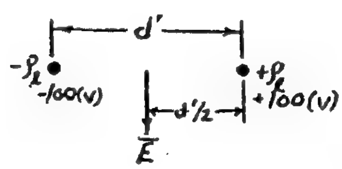
$$d = \frac{1}{2}\left(D + \sqrt{D^2 - 4a^2}\right) = \frac{1}{2}\left[2 + \sqrt{2^2 - 4(0.3)^2}\right] = 1.954 \text{ (cm)}.$$

$$d_i = D - d = 2 - 1.954 = 0.046 \text{ (cm)} = 0.46 \text{ (mm)}.$$

b)
$$\rho_\ell = \frac{2\pi\epsilon_0 V_1}{\ln(d/a)} = \frac{2\pi\left(\frac{1}{36\pi}\times 10^{-9}\right)\times 100}{\ln(1.954/0.3)} = 2.96\times 10^{-9} \text{ (F/m)}$$

$$= 2.96 \text{ (nF/m)}.$$

c) The equivalent line charges are separated by



$$d' = d - d_i = 1.954 - 0.046$$
$$= 1.908 \text{ (cm)}.$$

$$|\bar{E}| = \frac{\rho_\ell}{2\pi\epsilon_0 (d'/2)}\times 2 = 111.9 \text{ (V/m)},$$

— in a direction normal to the plane containing the wires.

Chapter 4

Steady Electric Currents

P.4-1 a) $R = \frac{\ell}{\sigma S} = \frac{V}{I} \longrightarrow \sigma = \frac{\ell I}{SV} = 3.54\times 10^{7}$ (S/m).

b) $E = \frac{V}{\ell} = 6\times 10^{-3}$ (V/m).

c) $P = VI = 1$ (W).

d) $\rho_e = -\frac{\sigma}{\mu_e}$. The given electron mobility 1.4×10^{-3} (m²·V/s) is that of a good conductor.

$$u = \left|\frac{J}{\rho_e}\right| = \left|\frac{\mu_e J}{\sigma}\right| = |\mu_e E| = 1.4\times 10^{-3}\times(6\times 10^{-3})$$
$$= 8.4\times 10^{-6} \text{ (m/s)}.$$

P.4-2 R_1 = Resistance per unit length of core $= \frac{1}{\sigma S_1} = \frac{1}{\sigma\pi a^2}$.

R_2 = Resistance per unit length of coating $= \frac{1}{0.1\sigma S_2}$.

Let b = Thickness of coating. $\longrightarrow S_2 = \pi(a+b)^2 - \pi a^2 = \pi(2ab+b^2)$.

a) $R_1 = R_2 \longrightarrow b = (\sqrt{11}-1)a = 2.32a$.

b) $I_1 = I_2 = \frac{I}{2}$. $\quad J_1 = \frac{I}{2\pi a^2}$, $\quad J_2 = \frac{I}{2S_2} = \frac{I}{20S_1} = \frac{I}{20\pi a^2}$.

$$E_1 = \frac{J_1}{\sigma} = \frac{I}{2\pi a^2\sigma},\quad E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2\sigma}.$$

Thus, $J_1 = 10J_2$ and $E_1 = E_2$.

P.4-3 $\rho_0 = \frac{Q_0}{(4\pi/3)b^3} = \frac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239$ (C/m³), $\quad \rho = \rho_0 e^{-(\sigma/\epsilon)t}$.

a) $R<b$: $\bar{E}_i = \bar{a}_R \frac{(4\pi/3)R^3\rho}{4\pi\epsilon R^2} = \bar{a}_R \frac{\rho_0 R}{3\epsilon} e^{-(\sigma/\epsilon)t} = \bar{a}_R\, 7.5\times 10^{9} R e^{-9.42\times 10^{11}t}$ (V/m).

$R>b$: $\bar{E}_o = \bar{a}_R \frac{Q_0}{4\pi\epsilon_0 R^2} = \bar{a}_R \frac{9}{R^2}\times 10^{6}$ (V/m).

b) $R<b$: $\bar{J}_i = \sigma\bar{E}_i = \bar{a}_R\, 7.5\times 10^{10} R e^{-9.42\times 10^{11}t}$ (A/m²).

$R>b$: $\bar{J}_o = 0$.

P.4-4 a) $e^{-(\sigma/\epsilon)t} = \frac{\rho}{\rho_0} = 0.01 \longrightarrow t = \frac{\ln 100}{(\sigma/\epsilon)} = 4.88\times10^{-12}\,(s) = 4.88\,(ps).$

b) $W_i = \frac{\epsilon}{2}\int_{V'} E_i^2\, dv' = \frac{2\pi\rho_0^2 b^5}{45\epsilon} e^{-2(\sigma/\epsilon)t} = (W_i)_0 \left[e^{-(\sigma/\epsilon)t}\right]^2.$

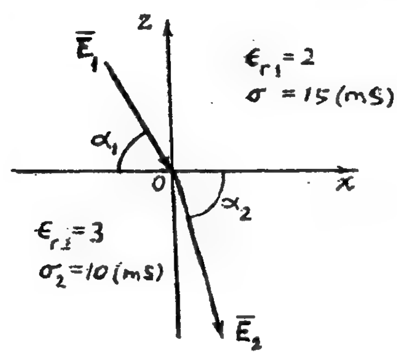
$\therefore \frac{W_i}{(W_i)_0} = \left[e^{-(\sigma/\epsilon)t}\right]^2 = 0.01^2 = 10^{-4}.$ Energy dissipated as heat loss.

c) Electrostatic energy stored outside the sphere $W_o = \frac{\epsilon_0}{2}\int_b^\infty E_o^2\, 4\pi R^2 dR = \frac{Q_0^2}{8\pi\epsilon_0 b} = 45\ (kJ)$
— Constant.

P.4-5 $I_1 = 0.1\,(A),\ P_{R1} = 3.33\,(mW);\quad I_2 = 0.02\,(A),\ P_{R2} = 8.00\,(mW);$
$I_3 = 0.0133\,(A),\ P_{R3} = 5.31\,(mW);\quad I_4 = 0.0333\,(A),\ P_{R4} = 8.87\,(mW);$
$I_5 = 0.0667\,(A),\ P_{R5} = 44.5\,(mW).\qquad \sum_n P_{Rn} = V_0 I_1 = 70\,(mW).$

Total resistance seen by the source $= 7\ (\Omega)$.

P.4-6



$\bar{E}_1 = \bar{a}_x 20 - \bar{a}_z 50\ \ (V/m).$

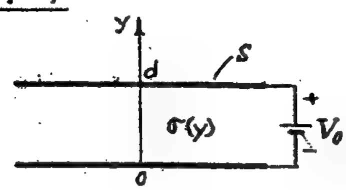
a) $E_{2t} = E_{1t} = 20.$
$J_{2n} = J_{1n} \longrightarrow \sigma_2 E_{2n} = \sigma_1 E_{1n}$
$\longrightarrow E_{2n} = \frac{\sigma_1}{\sigma_2} E_{1n} = \frac{15}{10}(-50) = -75.$
$\therefore \bar{E}_2 = \bar{a}_x 20 - \bar{a}_z 75\ \ (V/m).$

b) $\bar{J}_1 = \sigma_1 \bar{E}_1 = 15\times10^{-3}(\bar{a}_x 20 - \bar{a}_z 50) = \bar{a}_x 0.3 - \bar{a}_z 0.75\ \ (A/m^2).$
$\bar{J}_2 = \sigma_2 \bar{E}_2 = 10\times10^{-3}(\bar{a}_x 20 - \bar{a}_z 75) = \bar{a}_x 0.2 - \bar{a}_z 0.75\ \ (A/m^2).$

c) $\alpha_1 = \tan^{-1}\left(\frac{50}{20}\right) = 68.2°,\qquad \alpha_2 = \tan^{-1}\left(\frac{75}{20}\right) = 75.1°.$

d) $D_{2n} - D_{1n} = \rho_s \longrightarrow \epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \rho_s.$
$\rho_s = \epsilon_0(-3\times75 + 2\times50) = -125\,\epsilon_0 = -1.105\ (nC/m^2).$

P.4-7



$\sigma(y) = \sigma_1 + (\sigma_2 - \sigma_1)\frac{y}{d}$.

a) Neglecting fringing effect and assuming a current density:

$$\bar{J} = -\bar{a}_y J_0 \longrightarrow \bar{E} = \frac{\bar{J}}{\sigma} = -\bar{a}_y \frac{J_0}{\sigma(y)}.$$

$$V_0 = -\int_0^d \bar{E}\cdot\bar{a}_y\,dy = \int_0^d \frac{J_0\,dy}{\sigma_1 + (\sigma_2-\sigma_1)\frac{y}{d}} = \frac{J_0 d}{\sigma_2 - \sigma_1}\ln\frac{\sigma_2}{\sigma_1}.$$

$$R = \frac{V_0}{I} = \frac{V_0}{J_0 S} = \frac{d}{(\sigma_2-\sigma_1)S}\ln\frac{\sigma_2}{\sigma_1}.$$

b) $(\rho_s)_u = \epsilon_0 E_y(d) = \frac{\epsilon_0 J_0}{\sigma_2} = \frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_2 d\ln(\sigma_2/\sigma_1)}$ on upper plate,

$(\rho_s)_\ell = -\epsilon_0 E_y(0) = -\frac{\epsilon_0 J_0}{\sigma_1} = -\frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_1 d\ln(\sigma_2/\sigma_1)}$ on lower plate.

P.4-8 a) Continuity of the normal component of $\bar{J}$ assures the same current in both media. By Kirchhoff's voltage law:

$$V_0 = (R_1 + R_2)I = \left(\frac{d_1}{\sigma_1 S} + \frac{d_2}{\sigma_2 S}\right)I.$$

$$\therefore\quad J = \frac{I}{S} = \frac{V_0}{(d_1/\sigma_1)+(d_2/\sigma_2)} = \frac{\sigma_1\sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}.$$

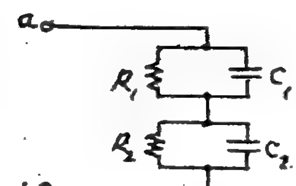
b) Two equations are needed for the determination of $\bar{E}_1$ and $\bar{E}_2$: $V_0 = E_1 d_1 + E_2 d_2$

and $\sigma_1 E_1 = \sigma_2 E_2$.

Solving, we have $E_1 = \dfrac{\sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$

and $E_2 = \dfrac{\sigma_1 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$.

c) Equivalent R-C circuit between terminals a and b:



$R_1 = \frac{d_1}{\sigma_1 S}$.

$R_2 = \frac{d_2}{\sigma_2 S}$.

$C_1 = \frac{\epsilon_1 S}{d_1}$.

$C_2 = \frac{\epsilon_2 S}{d_2}$.

P. 4-9 a) Same equivalent R-C circuit as that in Problem P.4-8 with

$$R_1 = \frac{1}{2\pi\sigma_1 L}\ln\left(\frac{c}{a}\right), \qquad R_2 = \frac{1}{2\pi\sigma_2 L}\ln\left(\frac{b}{c}\right).$$

$$C_1 = \frac{2\pi\epsilon_1 L}{\ln(c/a)}, \qquad C_2 = \frac{2\pi\epsilon_2 L}{\ln(b/c)}.$$

b) $$I = V_0 G = V_0\frac{1}{R_1+R_2} = \frac{2\pi\sigma_1\sigma_2 L V_0}{\sigma_1\ln(b/c)+\sigma_2\ln(c/a)}.$$

$$J_1 = J_2 = \frac{I}{2\pi r L} = \frac{\sigma_1\sigma_2 V_0}{r\,[\sigma_1\ln(b/c)+\sigma_2\ln(c/a)]}.$$

P. 4-10 Resistance $R = \frac{\ell}{\sigma S}$. (Eq. 4-16)

— Homogeneous material with a uniform cross section.

Between top and bottom flat faces: $S = \frac{\pi}{4}(b^2 - a^2)$.

$$R = \frac{4h}{\sigma\pi(b^2-a^2)}.$$

P. 4-11 Use Laplace's equation in cylindrical coordinates.

$$\bar{\nabla}^2 V = 0 \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0.$$

Solution: $V(r) = c_1 \ln r + c_2$.

Boundary conditions: $V(a) = V_0$; $\quad V(b) = 0$.

$$\longrightarrow V(r) = V_0\frac{\ln(b/r)}{\ln(b/a)}.$$

$$\bar{E}(r) = -\bar{a}_r\frac{\partial V}{\partial r} = \bar{a}_r\frac{V_0}{r\ln(b/a)}.$$

$$\bar{J}(r) = \sigma\bar{E}(r).$$

$$I = \int_S \bar{J}\cdot d\bar{s} = \int_0^{\pi/2}\bar{J}\cdot(\bar{a}_r h r\,d\phi) = \frac{\pi\sigma h V_0}{2\ln(b/a)}.$$

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi\sigma h}.$$

P. 4-12 Assume a potential difference V_0 between the inner and outer spheres.

$$\nabla^2 V = 0 \rightarrow \frac{1}{R^2}\frac{d}{dR}(R^2 V) = 0. \longrightarrow V = \frac{k}{R} \longrightarrow E_R = \frac{k}{R^2}.$$

$$V_0 = -\int_{R_2}^{R_1} E_R\, dR = -k\int_{R_2}^{R_1} \frac{1}{R^2}\, dR = k\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

$$\longrightarrow k = \frac{V_0}{\frac{1}{R_1} - \frac{1}{R_2}}. \qquad J_R = \sigma E_R = \frac{\sigma V_0}{\frac{1}{R_1} - \frac{1}{R_2}}.$$

$$I = \int_0^{2\pi}\int_0^{\pi} J_R R^2 \sin\theta\, d\theta\, d\phi = \frac{4\pi\sigma V_0}{\frac{1}{R_1} - \frac{1}{R_2}}.$$

$$R = \frac{V_0}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Chapter 5

Static Magnetic Fields

P. 5-1 $\bar{F} = Q(\bar{E} + \bar{u}\times\bar{B}) = 0.$

$$\bar{E} = -\bar{u}\times\bar{B} = -\bar{a}_x u_0 \times (\bar{a}_x B_x + \bar{a}_y B_y + \bar{a}_z B_z)$$
$$= u_0(\bar{a}_y B_z - \bar{a}_z B_y).$$

P. 5-2 $\bar{B} = \bar{a}_\phi B_\phi = \bar{a}_\phi \frac{\mu_0 N I}{2\pi r}.$

$$\Phi = \int_S B_\phi\, ds = \frac{\mu_0 N I}{2\pi}\int_a^b \frac{h}{r}\,dr$$
$$= \frac{\mu_0 N I h}{2\pi}\ln\frac{b}{a}.$$

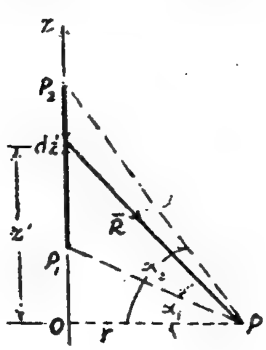
If B_ϕ at $r = \frac{a+b}{2}$ is used, $\Phi' = \frac{\mu_0 N I h}{\pi}\left(\frac{b-a}{b+a}\right).$

$$\%\ \text{error} = \frac{\Phi' - \Phi}{\Phi}\times 100$$
$$= \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1\right]\times 100.$$

For $\frac{b}{a} = 5$, the error is $\left[\frac{2(5-1)}{(5+1)\ln 5} - 1\right]\times 100,$

or $-17.2\ \%$ (too low).

P. 5-3 a) Use Eq. (5-32c). $d\bar{\ell}' = \bar{a}_z dz'$, $\bar{R} = \bar{a}_r r - \bar{a}_z z'$.



$$d\bar{\ell}'\times\bar{R} = \bar{a}_z dz'\times(\bar{a}_r r - \bar{a}_z z') = \bar{a}_\phi r\, dz'.$$
$$\bar{B}_P = \bar{a}_\phi \frac{\mu_0 I}{4\pi}\int \frac{r\,dz'}{(z'^2 + r^2)^{3/2}}.$$

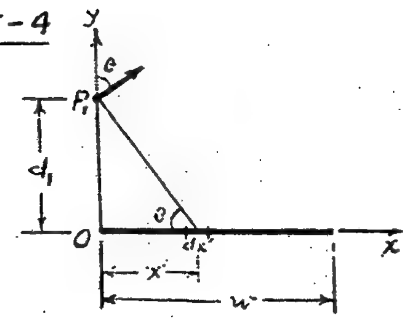
Let $z' = r\tan\alpha$, $dz' = r\sec^2\alpha\, d\alpha$.

$$\bar{B}_P = \bar{a}_\phi \frac{\mu_0 I}{4\pi r}\int_{\alpha_1}^{\alpha_2}\cos\alpha\, d\alpha$$
$$= \bar{a}_\phi \frac{\mu_0 I}{4\pi r}(\sin\alpha_2 - \sin\alpha_1).$$

b) For an infinitely long wire: $\alpha_2 \to 90°$ and $\alpha_1 \to -90°$.

$\bar{B}_P$ becomes $\bar{a}_\phi \frac{\mu_0 I}{2\pi r}$, as in Eq. (5-35).

P. 5-4



Use Eq. (5-35):

$$d\bar{B}_{P_1} = \bar{a}_x dB_x + \bar{a}_y dB_y$$

$$= \bar{a}_x (dB_{P_1}) \sin\theta + \bar{a}_y (dB_{P_1}) \cos\theta,$$

Where $dB_{P_1} = \dfrac{\mu_0 (I/w)\, dx'}{2\pi (x'^2 + d_1^2)^{1/2}}$,

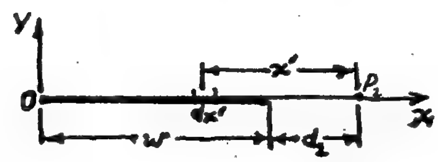
$$\sin\theta = \frac{d}{(x'^2+d^2)^{1/2}}, \quad \cos\theta = \frac{x'}{(x'^2+d^2)^{1/2}}.$$

$$\bar{B}_{P_1} = \bar{a}_x B_x + \bar{a}_y B_y,$$

where $B_x = \dfrac{\mu_0 I d}{2\pi w} \displaystyle\int_0^w \frac{dx'}{x'^2+d^2} = \frac{\mu_0 I}{2\pi w} \tan^{-1}\left(\frac{w}{d}\right)$,

and $B_y = \dfrac{\mu_0 I}{2\pi w} \displaystyle\int_0^w \frac{x'\,dx'}{x'^2+d^2} = \frac{\mu_0 I}{4\pi w} \ln\left(1+\frac{w}{d}\right)$.

P. 5-5

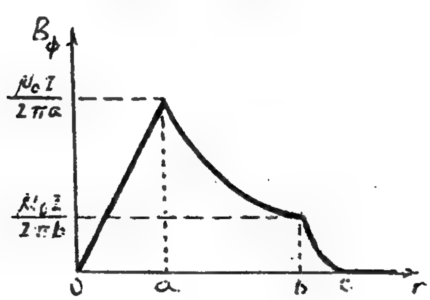


I flows into the paper (in $-\bar{a}_z$ direction).

$$d\bar{B}_{P_2} = -\bar{a}_z \frac{\mu_0 I\, dx'}{2\pi w x'}.$$

$$\bar{B}_{P_2} = -\bar{a}_z \frac{\mu_0 I}{2\pi w} \int_{d_2}^{d_2+w} \frac{dx'}{x'} = -\bar{a}_z \frac{\mu_0 I}{2\pi w} \ln\left(1+\frac{w}{d_2}\right).$$

P. 5-6 Apply Ampère's circuital law, Eq. (5-10), and assume the medium to be nonmagnetic:



$$\oint \bar{B} \cdot d\bar{\ell} = \mu_0 I.$$

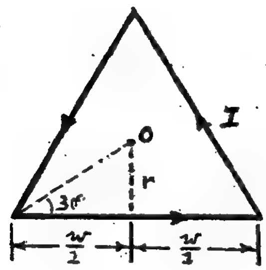
For $0 \le r \le a$, $\bar{B} = \bar{a}_\phi \dfrac{\mu_0 r I}{2\pi a^2}$.

For $a \le r \le b$, $\bar{B} = \bar{a}_\phi \dfrac{\mu_0 I}{2\pi r}$.

For $b \le r \le c$, $\bar{B} = \bar{a}_\phi \left(\dfrac{c^2-r^2}{c^2-b^2}\right) \dfrac{\mu_0 I}{2\pi r}$.

For $r \ge c$, $\bar{B} = 0$.

P. 5-7



Assume that the current flows in the counterclockwise direction in a triangle lying in the xy-plane. From Eq. (5-34) and noting that

$$L = \frac{w}{2} \text{ and } r = \frac{w}{2}\tan 30° = \frac{w}{2\sqrt{3}},$$

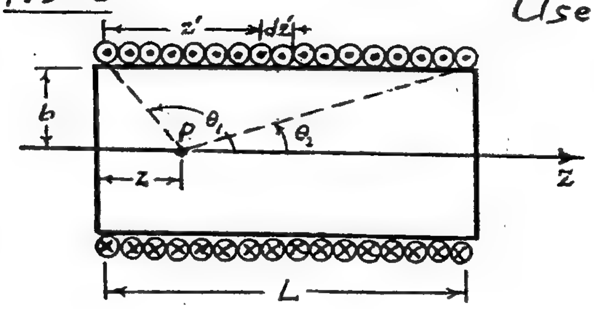
we have

$$\bar{B} = 3\left(\bar{a}_z \frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}\right) \text{ at } O.$$

$$L/r = \sqrt{3}, \quad \sqrt{L^2+r^2} = \frac{w}{\sqrt{3}}.$$

$$\bar{B} = \bar{a}_z \frac{3\mu_0 I}{2\pi} \frac{\sqrt{3}}{w/\sqrt{3}} = \bar{a}_z \frac{9\mu_0 I}{2\pi w}.$$

P. 5-8



Use Eq. (5-37):

$$d\bar{B} = \bar{a}_z \frac{\mu I b^2}{2[(z'-z)^2+b^2]^{3/2}}\left(\frac{N}{L}\right)dz',$$

$$\bar{B} = \bar{a}_z \frac{\mu N I b^2}{2L}$$

$$= \bar{a}_z \frac{\mu N I}{2L}\left[\frac{L-z}{\sqrt{(L-z)^2+b^2}} - \frac{z}{\sqrt{z^2+b^2}}\right]$$

$$= \bar{a}_z \frac{\mu N I}{2L}(\cos\theta_2 - \cos\theta_1).$$

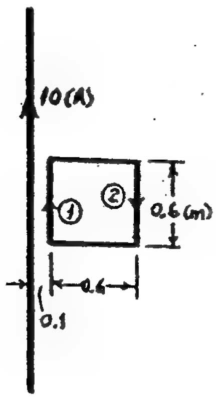
For $L \to \infty$, $\theta_2 \to 0$ and $\theta_1 \to \pi$, and $\bar{B} \to \bar{a}_z \frac{\mu N I}{L} = \bar{a}_z \mu n I$.

P. 5-9 a) $\bar{B} = \bar{\nabla}\times\bar{A} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r} = -\bar{a}_\phi \frac{\partial A_z}{\partial r}$. (No change with z.)

$$\frac{dA_z}{dr} = -\frac{\mu_0 I}{2\pi r} \longrightarrow A_z = -\frac{\mu_0 I}{2\pi}\ln r + c.$$

$$A_z = 0 \text{ at } r = r_0 \longrightarrow c = \frac{\mu_0 I}{2\pi}\ln r_0.$$

$$\therefore \quad \bar{A} = \bar{a}_z A_z = \bar{a}_z \frac{\mu_0 I}{2\pi}\ln\left(\frac{r_0}{r}\right).$$



b) Use $\Phi = \oint \bar{A} \cdot d\bar{\ell}$.

Horizontal sides have no effect.

Side ①: $\int \bar{A} \cdot d\bar{\ell} = \left(\frac{\mu_0 I}{2\pi} \ln \frac{r_0}{0.1}\right) \times 0.6$.

Side ③: $\int \bar{A} \cdot d\bar{\ell} = -\left(\frac{\mu_0 I}{2\pi} \ln \frac{r_0}{0.7}\right) \times 0.6$.

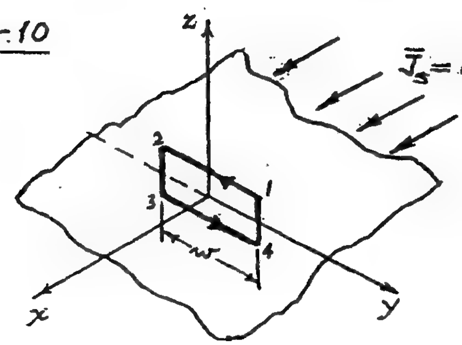
$$\oint \bar{A} \cdot d\bar{\ell} = \left(\frac{\mu_0 I}{2\pi} \ln \frac{0.7}{0.1}\right) \times 0.6$$

$$= \frac{(4\pi 10^{-7}) \times 10 \times 0.6}{2\pi} \ln 7$$

$$= 2.34 \times 10^{-6} \text{ (Wb)}.$$

$\therefore \ \Phi = 2.34 \ (\mu\text{Wb})$.

P.5-10



Infinite current sheet $\longrightarrow \bar{B}$ antisymmetrical and independent of x and y.

a) Apply Ampère's circuital law to path 12341:

$$\oint_C \bar{B} \cdot d\bar{\ell} = \mu_0 I \longrightarrow 2wB_y = \mu_0 J_{s0} w.$$

$$\longrightarrow B_y = \begin{cases} -\mu_0 J_{s0}/2 \text{ at } (0,0,z), \\ +\mu_0 J_{s0}/2 \text{ at } (0,0,-z). \end{cases}$$

or, $\quad \bar{B} = \frac{\mu_0}{2} \bar{J}_s \times \bar{a}_n$.

b) For $z > 0$, $\quad \bar{\nabla} \times \bar{A} = \bar{B} = \bar{a}_y\left(-\frac{\mu_0 J_{s0}}{2}\right)$.

$\bar{A}$ is independent of x and y.

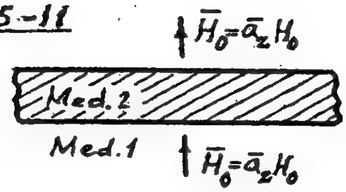
$$\frac{dA_x}{dz} = -\frac{\mu_0 J_{s0}}{2}.$$

$$A_x = -\frac{\mu_0 J_{s0}}{2} z + c.$$

At $z = z_0$, $A_x = 0 = -\frac{\mu_0 J_{s0}}{2} z_0 + c \longrightarrow c = \frac{\mu_0 J_{s0}}{2} z_0$.

$$\therefore \ \bar{A} = -\frac{\mu_0}{2}(z - z_0)\bar{J}_s.$$

P. 5-11



a) Given $\bar{B}_2 = \mu_2 \bar{H}_2$.

$B_{2z} = B_{1z} \longrightarrow \mu H_2 = \mu_0 H_0 \longrightarrow \bar{H}_2 = \bar{a}_z H_2 = \bar{a}_z \frac{\mu_0}{\mu} H_0$.

b) Given $\bar{B}_2 = \mu_0 (\bar{H}_2 + \bar{M}_i)$.

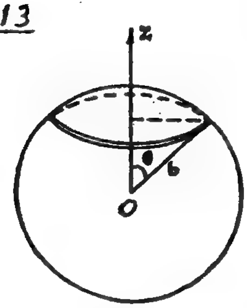
$B_{2z} = B_{1z} \longrightarrow \mu_0 (H_2 + M_i) = \mu_0 H_0 \longrightarrow \bar{H}_2 = \bar{a}_z (H_0 - M_i)$.

P. 5-12 a) $r < a$: $\left.\begin{array}{l}\bar{H} = \bar{a}_z nI, \\ \bar{B} = \bar{a}_z \mu nI\end{array}\right\}$ Eq. (6-14). $\quad$ $a < r < b$: $\bar{H} = \bar{a}_z nI$, $\bar{B} = \bar{a}_z \mu_0 nI$, $\bar{M} = 0$.

$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z \left(\frac{\mu}{\mu_0} - 1\right) nI$.

b) $\bar{J}_m = \bar{\nabla} \times \bar{M} = 0$; $\quad \bar{J}_{ms} = \bar{M} \times \bar{a}_n = (\bar{a}_z \times \bar{a}_r)\left(\frac{\mu}{\mu_0} - 1\right) nI = \bar{a}_\phi \left(\frac{\mu}{\mu_0} - 1\right) nI$.

P. 5-13



$\bar{M} = \bar{a}_z M_0$

a) $\bar{J}_m = \bar{\nabla} \times \bar{M} = 0$.

$\bar{J}_{ms} = (\bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta) M \times \bar{a}_R$

$= \bar{a}_\phi M_0 \sin\theta$.

b) Apply Eq. (5-37) to a loop of radius $b \sin\theta$ carrying a current $J_{ms}\, b\, d\theta$:

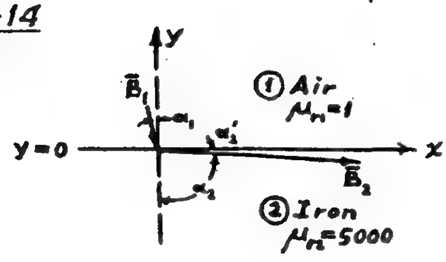
$$d\bar{B} = \bar{a}_z \frac{\mu_0 (J_{ms} b\, d\theta)(b \sin\theta)^2}{2 (b^2)^{3/2}}$$

$$= \bar{a}_z \frac{\mu_0 M_0}{2} \sin^3\theta .$$

$$\bar{B} = \int d\bar{B} = \bar{a}_z \frac{\mu_0 M_0}{2} \int_0^\pi \sin^3\theta\, d\theta = \bar{a}_z \frac{2}{3} \mu_0 M_0 = \frac{2}{3} \mu_0 \bar{M},$$

at the center O.

P. 5-14



a) $\bar{B}_1 = \bar{a}_x 2 - \bar{a}_y 10$ (mT),

$\bar{B}_2 = \bar{a}_x B_{2x} - \bar{a}_y B_{2y}$.

$H_{2x} = \frac{B_{2x}}{5000 \mu_0} = H_{1x} = \frac{2}{\mu_0}$

$\longrightarrow B_{2x} = 10{,}000$ (mT),

$B_{2y} = B_{1y} = -10$ (mT).

$\therefore \bar{B}_2 = \bar{a}_x 10{,}000 - \bar{a}_y 10$ (mT).

$$\tan\alpha_2 = \frac{\mu_2}{\mu_1} \tan\alpha_1 = 5000 \left(\frac{B_{1x}}{B_{1y}}\right) = 1{,}000 \longrightarrow \alpha_2 = 89.94^\circ,\ \alpha_2' = 0.04^\circ.$$

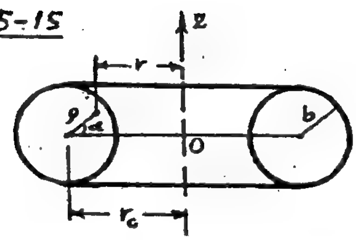
b) If $\bar{B}_2 = \bar{a}_x 10 + \bar{a}_y$ (mT), $\bar{B}_1 = \bar{a}_x B_{1x} + \bar{a}_y B_{1y}$.

$$H_{1x} = \frac{B_{1x}}{\mu_1} = H_{2x} = \frac{B_{2x}}{\mu_2} \longrightarrow B_{1x} = \frac{1}{\mu_{r2}} B_{2x} = \frac{10}{5000} = 0.002.$$

$$B_{1y} = B_{2y} = 2. \qquad \therefore \bar{B}_1 = \bar{a}_x 0.002 + \bar{a}_y 2 \quad (\text{mT}).$$

$$\alpha_1 = \tan^{-1}\frac{B_{1x}}{B_{1y}} \cong \frac{0.002}{2} = 0.001 \ (\text{rad}) = 0.057°$$

P. 5-15



$$\bar{B} = \bar{a}_\phi B_\phi = \bar{a}_\phi \frac{\mu_0 NI}{2\pi r}, \quad r = r_0 - \rho\cos\alpha.$$

$$\Phi = \frac{\mu_0 NI}{2\pi}\int_0^b\int_0^{2\pi}\frac{\rho\, d\alpha\, d\rho}{r_0 - \rho\cos\alpha} = \mu_0 NI\left(r_0 - \sqrt{r_0^2 - b^2}\right).$$

$$\therefore L = \frac{N\Phi}{I} = \mu_0 N^2\left(r_0 - \sqrt{r_0^2 - b^2}\right).$$

If $r_0 \gg b$, $B_\phi \cong \frac{\mu_0 NI}{2\pi r_0}$ (constant).

$$\Phi \cong B_\phi S = B_\phi(\pi b^2) = \frac{\mu_0 N b^2 I}{2 r_0} \longrightarrow L \cong \frac{\mu_0 N^2 b^2}{2 r_0}.$$

P. 5-16 For I in the long straight wire, $\bar{B} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}$.

$$\Lambda_{12} = \int_S \bar{B}\cdot d\bar{s} = \int B_\phi \frac{2}{\sqrt{3}}(r-d)\,dr = \frac{\mu_0 I}{\pi\sqrt{3}}\int_d^{d+\frac{\sqrt{3}}{2}b}\left(\frac{r-d}{r}\right)dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\left(1 + \frac{\sqrt{3}b}{2d}\right)\right],$$

$$L_{12} = \frac{\Lambda_{12}}{I} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1 + \frac{\sqrt{3}b}{2d}\right)\right].$$

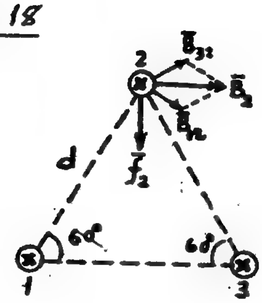
P. 5-17 Approximate the magnetic flux due to the long loop linking with the small loop by that due to two infinitely long wires carrying equal and opposite current I.

$$\Lambda_{12} = \frac{\mu_0 h_2 I}{2\pi}\int_0^{w_2}\left(\frac{1}{d+x} - \frac{1}{w_1+d+x}\right)dx$$

$$= \frac{\mu_0 h_2 I}{2\pi}\ln\left(\frac{w_2+d}{d}\cdot\frac{w_1+d}{w_1+w_2+d}\right).$$

$$L_{12} = \frac{\Lambda_{12}}{I} = \frac{\mu_0 h_2}{2\pi}\ln\frac{(w_1+d)(w_2+d)}{d(w_1+w_2+d)}.$$

P.5-18



$I_1 = I_2 = I_3 = 25$ (A) ; $d = 0.15$ (m).

$\bar{B}_2 = \bar{a}_x 2B_{12}\cos 30° = \bar{a}_x \frac{\sqrt{3}\,\mu_0 I}{2\pi d}$.

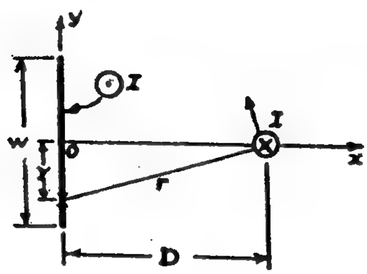
Force per unit length on wire 2 :

$$\bar{f}_2 = -\bar{a}_y I B_2 = -\bar{a}_y \frac{\sqrt{3}\,\mu_0 I^2}{2\pi d}$$

$$= -\bar{a}_y 1150\mu_0 = -\bar{a}_y 1.44\times 10^{-3}\ \text{(N/m)}.$$

Forces on all three wires are of equal magnitude and toward the center of the triangle.

P.5-19 Magnetic field intensity at the wire due to the current $dI = \frac{I}{w}dy$ in an elemental dy is



$$|d\bar{H}| = \frac{dI}{2\pi r} = \frac{I\,dy}{2\pi w\sqrt{D^2+y^2}}.$$

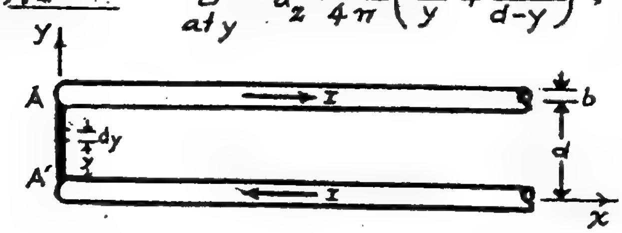
Symmetry ⟶ $\bar{H}$ at the wire has only a y-component.

$$\bar{H} = \bar{a}_y \int (dH)\cdot\left(\frac{D}{r}\right) = \bar{a}_y 2\int_0^{w/2} \frac{ID\,dy}{2\pi w(D^2+y^2)}$$

$$= \bar{a}_y \frac{I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right).$$

$$\bar{f}' = \bar{I}\times\bar{B} = (-\bar{a}_z I)\times(\mu_0 \bar{H}) = \bar{a}_x \frac{\mu_0 I^2}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right)\quad \text{(N/m)}.$$

P.5-20 $\underset{\text{at } y}{\bar{B}} = -\bar{a}_z \frac{\mu_0 I}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)$, $d\bar{\ell} = \bar{a}_y dy$.



(A rail-gun problem.)

$$d\bar{F} = I\,d\bar{\ell}\times\bar{B}$$

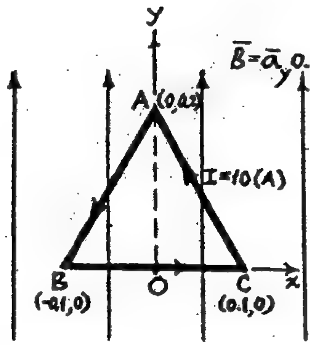
$$= -\bar{a}_x \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)dy$$

$$\bar{F} = -\bar{a}_x \frac{\mu_0 I^2}{4\pi}\int_b^{d-b}\left(\frac{1}{y} + \frac{1}{d-y}\right)dy$$

$$= -\bar{a}_x \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{d}{b} - 1\right).$$

P.5-21 Force in a uniform magnetic field:

$$\bar{F} = I\bar{L} \times \bar{B} = -\bar{B} \times (I\bar{L}).$$



$\bar{B} = \bar{a}_y 0.5\,(T)$. $-\bar{B}\times$	$I(\overrightarrow{AB})$	$I(\overrightarrow{CA})$	$I(\overrightarrow{BC})$
$-(\bar{a}_y 0.5)\times$	$10(-\bar{a}_x 0.1 - \bar{a}_y 0.2)$,	$10(-\bar{a}_x 0.1 + \bar{a}_y 0.2)$,	$10\bar{a}_x 0.2$
Force:	$-\bar{a}_z 0.5$	$-\bar{a}_z 0.5$	$\bar{a}_z 1.0$ (N)

Torque on loop:

$$\bar{T} = \bar{m} \times \bar{B} = (\bar{a}_z I S) \times \bar{B}$$

$$= (\bar{a}_z 10 \times \tfrac{1}{2} \times 0.2 \times 0.2) \times (\bar{a}_y 0.5) = -\bar{a}_x 0.1\ (\text{N}\cdot\text{m}).$$

P.5-22 $\bar{B}_2$ at the center of the large circular turn of wire carrying a current I_2 is (by setting $z = 0$ in Eq. 5-37):

$$\bar{B}_2 = \bar{a}_{n2} \frac{\mu_0 I_2}{2 r_2}.$$

Torque on the small circular turn of wire carrying a current I_1 is

$$\bar{T} = \bar{m}_1 \times \bar{B}_2 \cong (\bar{a}_{n1} I_1 \pi r_1^2) \times \left(\bar{a}_{n2} \frac{\mu_0 I_2}{2 r_2}\right)$$

$$= (\bar{a}_{n1} \times \bar{a}_{n2}) \frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2},$$

which is a torque having a magnitude $\frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2} \sin\theta$ and a direction tending to align the magnetic fluxes produced by I_1 and I_2.

Chapter 6

Time-Varying Fields and Maxwell's Equations

P.6-1

$$\mathcal{V} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$
$$= -\int_S \frac{\partial}{\partial t}(\bar{\nabla}\times\bar{A})\cdot d\bar{s}$$
$$= -\oint \frac{\partial \bar{A}}{\partial t}\cdot d\bar{\ell}.$$

P.6-2

$$\bar{B} = \bar{a}_z\, 3\cos(5\pi 10^7 t - \tfrac{1}{3}\pi y)\times 10^{-6}\ (T).$$
$$\int_S \bar{B}\cdot d\bar{s} = \int_0^{0.3} \bar{a}_z\, 3\cos(5\pi 10^7 t - \tfrac{1}{3}\pi y)10^{-6}\cdot(\bar{a}_z 0.1\, dy)$$
$$= -\frac{0.9}{\pi}\left[\sin(5\pi 10^7 t - 0.1\pi) - \sin 5\pi 10^7 t\right]\times 10^{-6}\ (Wb).$$
$$\mathcal{V} = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s} = 4.5\left[\cos(5\pi 10^7 t - 0.1\pi) - \cos 5\pi 10^7 t\right]\ (V).$$
$$i = \frac{\mathcal{V}}{2R} = 0.15\left[\cos(5\pi 10^7 t - 0.1\pi) - \cos 5\pi 10^7 t\right]$$
$$= 0.023\sin(5\pi 10^7 t - 0.05\pi)\quad (A)$$
$$= 23\sin(5\pi 10^7 t - 9^\circ)\quad (mA).$$

P.6-3 Using phasors with a sine reference:

$$\bar{B}_1 = \bar{a}_\phi \frac{\mu_0 I_1}{2\pi r} \longrightarrow \Phi_{12} = \int_{S_2} \bar{B}_1\cdot d\bar{s}_2 = \frac{\mu_0 I_1 h}{2\pi}\int_d^{d+w}\frac{dr}{r}$$
$$v_2 = -\frac{d\Phi_{12}}{dt} \longrightarrow \text{Phasors: } V_2 = -j\omega\Phi_{12}; \quad = \frac{\mu_0 I_1 h}{2\pi}\ln\left(1+\frac{w}{d}\right).$$
$$I_2 = \frac{V_2}{R+j\omega L} = -\frac{j\omega\mu_0 I_1 h}{2\pi(R+j\omega L)}\ln\left(1+\frac{w}{d}\right)$$
$$= -\frac{\omega\mu_0 I_1 h}{2\pi(\omega L - jR)}\ln\left(1+\frac{w}{d}\right) = -\frac{\omega\mu_0 I_1 h}{2\pi\sqrt{R^2+\omega^2L^2}}\ln\left(1+\frac{w}{d}\right)e^{j\tan^{-1}(R/\omega L)}$$
$$\longrightarrow i_2 = -\frac{\omega\mu_0 I_1 h}{2\pi\sqrt{R^2+\omega^2L^2}}\ln\left(1+\frac{w}{d}\right)\sin\left(\omega t + \tan^{-1}\frac{R}{\omega L}\right).$$

P. 6-4 $\quad \bar{B}_1 = -\bar{a}_x \dfrac{\mu_0 I_0}{2\pi r}$.

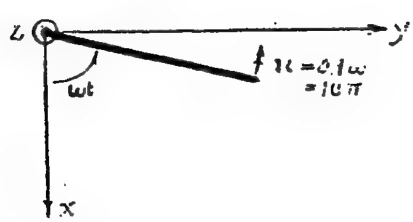
Induced emf in loop $= \oint (\bar{u}_2 \times \bar{B}_1) \cdot d\bar{\ell}_2$.

$$\longrightarrow \mathcal{V}_2 = \frac{\mu_0 I_0 h u_0}{2\pi}\left(\frac{1}{d} - \frac{1}{d+w}\right),$$

in a clockwise direction.

$$i_2 = -\frac{\mathcal{V}_2}{R} = -\frac{\mu_0 I_0 h u_0 w}{2\pi d(d+w)}.$$

P. 6-5



a) If $L = 0$:

$$i = \frac{1}{R}(\bar{u} \times \bar{B}) \cdot (-\bar{a}_z 0.1)$$

$$= \frac{1}{0.5}(10\pi \times 0.04) \times 0.1 \sin \omega t$$

$$= 0.251 \sin 100\pi t \quad (A).$$

b) If $L = 0.0035$ (H):

$$\omega L = 100\pi \times 0.0035 = 1.1\ (\Omega),$$

$$\frac{1}{R + j\omega L} = \frac{1}{0.5 + j1.1} = \frac{1}{1.208\angle 65.6^\circ}.$$

$$\longrightarrow i_2 = \frac{(10\pi \times 0.04) \times 0.1}{1.208} \sin(\omega t - 65.6^\circ)$$

$$= 0.104 \sin(100\pi t - 65.6^\circ) \quad (A).$$

P. 6-6 $\quad \Phi = \bar{B}(t) \cdot \bar{S}(t) = -5 \cos \omega t \times 0.2\,(0.7 - x)$

$$= -\cos \omega t\,[0.7 - 0.35(1 - \cos \omega t)]$$

$$= -0.35 \cos \omega t\,(1 + \cos \omega t) \quad (mT).$$

$$i = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}\,0.35\omega(\sin \omega t + \sin 2\omega t)$$

$$= -1.75\,\omega(\sin \omega t + \sin 2\omega t)$$

$$= -1.75\,\omega \sin \omega t\,(1 + 2\cos \omega t) \quad (mA).$$

P. 6-7 Conduction current density: σE.

Displacement current density: $j\omega D = j\omega\epsilon_0\epsilon_r E$.

For equal magnitude: $\sigma = 2\pi\epsilon_0\epsilon_r f$,

$$\text{or} \quad f = \frac{\sigma}{2\pi(\epsilon_0\epsilon_r)} = 18\times10^9\left(\frac{\sigma}{\epsilon_r}\right) \quad (Hz).$$

a) Seawater: $f = 18\times10^9\left(\frac{4}{72}\right) = 10^9\,(Hz) = 1\,(GHz)$.

b) Moist soil: $f = 18\times10^9\left(\frac{10^{-3}}{2.5}\right) = 7.2\times10^6\,(Hz)$, or $7.2\,(MHz)$.

P. 6-8

a) $$\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\epsilon}{\sigma} = \frac{(2\pi\times100\times10^9)\times\frac{1}{36\pi}\times10^{-9}}{5.70\times10^7}$$
$$= 9.75\times10^{-8}.$$

b) In a source-free conductor:

$$\bar{\nabla}\times\bar{H} = \sigma\bar{E}, \qquad (1)$$
$$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}. \qquad (2)$$
$$\bar{\nabla}\times(1):\ \bar{\nabla}\times\bar{\nabla}\times\bar{H} = \bar{\nabla}(\bar{\nabla}\cdot\bar{H}) - \bar{\nabla}^2\bar{H} - \sigma\bar{\nabla}\times\bar{E}. \qquad (3)$$

But $\bar{\nabla}\cdot\bar{H}=0$, Eq. (3) becomes

$$\bar{\nabla}^2\bar{H} + \sigma\bar{\nabla}\times\bar{E} = 0. \qquad (4)$$

Combining (2) and (4):

$$\bar{\nabla}^2\bar{H} - j\omega\mu\sigma\bar{H} = 0.$$

P. 6-9 $\bar{H}_1 = \bar{a}_x30 + \bar{a}_y40 + \bar{a}_z20$.

$B_{2n} = B_{1n} \longrightarrow H_{2z} = \frac{1}{\mu_{r2}}H_{1z} = 10.$

$\longrightarrow B_{2z} = B_{1z} = 20\mu_0.$

$\bar{a}_{n2}\times(\bar{H}_1 - \bar{H}_2) = \bar{J}$

$\longrightarrow \bar{a}_z\times(\bar{a}_x30 + \bar{a}_y40 - \bar{H}_2) = \bar{a}_x5.$

$\longrightarrow H_{2x} = 30,\quad H_{2y} = 45.$

a) $\bar{H}_2 = \bar{a}_x30 + \bar{a}_y45 + \bar{a}_z10$ (A/m). b) $\bar{B}_2 = 2\mu_0\bar{H}_2$ (T).

c) $\alpha_1 = \tan^{-1}\frac{\sqrt{30^2+40^2}}{20} = 68.2°$. d) $\alpha_2 = \tan^{-1}\frac{\sqrt{30^2+45^2}}{10} = 79.5°$.

P. 6-10 Medium 1: Free space.
Medium 2: $\mu_2 \to \infty$. H_2 must be zero so that B_2 is not infinite.

Boundary Conditions: $\bar{a}_{n2} \times \bar{H}_1 = \bar{J}_s$, $B_{1n} = B_{2n}$.

$E_{1t} = E_{2t}$, $\bar{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$.

P. 6-13 $E(z,t) = 50 \cos(2\pi 10^9 t - kz)$ (V/m) in air.

$E_0 = 50$ (V/m).

$f = 10^9$ (Hz), $T = \frac{1}{f} = 10^{-9}$ (s),

$\lambda = \frac{c}{f} = \frac{3\times 10^8}{10^9} = 0.3$ (m),

$k = \frac{2\pi}{\lambda} = \frac{20}{3}\pi$.

a) At $z = 100.125\lambda$, $kz = 200.25\pi$, which is same as for $kz = 0.25\pi$, or $\pi/4$.

It is a plot of $E(t) = 50 \cos 2\pi 10^9 (t - T/8)$ (V/m).

b) At $z = -100.125\lambda$, it is a plot of

$$E(t) = 50 \cos 2\pi 10^9 (t + T/8) \quad \text{(V/m)}.$$

c) At $t = T/4$, we plot versus z the following sinusoidal function:

$$E(z, \tfrac{T}{4}) = 50 \cos\left(-kz + \tfrac{\omega T}{4}\right) = 50 \cos\left[-k\left(z - \tfrac{\lambda}{4}\right)\right]$$
$$= 50 \cos \tfrac{20\pi}{3}(z - 0.075) \quad \text{(V/m)}.$$

P. 6-14 Use phasors and cosine reference.

$\bar{E} = \bar{a}_x E_0 e^{j\psi}$; $\bar{E}_1 = \bar{a}_x 0.03 e^{-j\pi/2}$; $\bar{E}_2 = \bar{a}_x 0.04 e^{-j\pi/3}$.

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \bar{a}_x \left[0.03 e^{-j\pi/2} + 0.04 e^{-j\pi/3}\right]$$
$$= \bar{a}_x \left[-j0.03 + 0.04\left(\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right)\right] = \bar{a}_x 0.068 e^{-j72.8^\circ}$$

$\longrightarrow E_0 = 0.068$ (V/m), $\psi = -72.8^\circ$.

P.6-15 Use phasors and cosine reference.

$$\bar{H} = \bar{a}_\phi H_0, \quad \bar{H}_1 = \bar{a}_\phi 10^{-4} e^{-j\pi/2}, \quad \bar{H}_2 = \bar{a}_\phi 2\times 10^{-4} e^{j\alpha}.$$

$$\longrightarrow H_0 = 10^{-4}(-j + 2e^{j\alpha})$$

$$= 10^{-4}\left[2\cos\alpha + j(2\sin\alpha - 1)\right].$$

$$2\sin\alpha - 1 = 0 \longrightarrow \alpha = 30°, \text{ or } \pi/6 \text{ (rad.)}$$

$$H_0 = 2\times 10^{-4}\cos 30° = 1.73\times 10^{-4} \quad (\text{A/m}).$$

P.6-17 See Section 10-2, pp. 428-429, Eqs. (10-6) and (10-7).

P.6-18 a) $k = \dfrac{\omega}{c} = \dfrac{2\pi(60\times 10^6)}{3\times 10^8} = 0.4\pi$ (rad/m).

b)
$$\bar{H} = \frac{1}{-j\omega\mu_0}\bar{\nabla}\times\bar{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & 0 & 0 \end{vmatrix}$$

$$= \frac{j}{\omega\mu_0}\bar{a}_\phi\frac{\partial E_r}{\partial z} = \bar{a}_\phi\frac{j}{\omega\mu_0}(-jk)\frac{E_0}{r}e^{-jkz}$$

$$= \bar{a}_\phi\frac{k}{\omega\mu_0}\frac{E_0}{r}e^{-jkz} = \bar{a}_\phi\frac{E_0}{120\pi r}e^{-j0.4\pi z} \quad (\text{A/m}).$$

c)
$$\bar{J}_s\Big|_{r=a} = \bar{a}_z H_\phi\Big|_{r=a} = \bar{a}_z\frac{E_0}{120\pi a}e^{-j0.4\pi z} \quad (\text{A/m}).$$

$$\bar{J}_s\Big|_{r=b} = -\bar{a}_z H_\phi\Big|_{r=b} = -\bar{a}_z\frac{E_0}{120\pi b}e^{-j0.4\pi z} \quad (\text{A/m}).$$

P.6-19 $k = \frac{\omega}{c} = \frac{2\pi\times10^9}{3\times10^8} = \frac{20\pi}{3}$ (rad/m).

$$\bar{H} = \frac{j}{\omega\mu_0}\bar{\nabla}\times\bar{E}.$$

In phasor form: $\bar{E} = \bar{a}_\theta \frac{10^{-3}}{R}\sin\theta\, e^{-jkR}$.

From Eq. (2-99):
$$\bar{H} = \frac{j}{\omega\mu_0}\frac{1}{R^2\sin\theta}\begin{vmatrix} \bar{a}_R & \bar{a}_\theta R & \bar{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 10^{-3}\sin\theta\cdot e^{-jkR} & 0 \end{vmatrix}$$
$$= \bar{a}_\phi \frac{10^{-3}}{\eta_0 R}\sin\theta\cdot e^{-jkR}.$$

In instantaneous form:
$$\bar{H}(R,\theta;t) = \bar{a}_\phi \frac{10^{-3}}{120\pi R}\sin\theta\cos(2\pi10^9 t - 20\pi R/3)\ \text{(A/m)}.$$

P.6-20 In phasor form:
$$\bar{E} = \bar{a}_y\, 0.1\sin(10\pi x)\, e^{-j\beta z}. \qquad (1)$$
$$\bar{H} = -\frac{1}{j\omega\mu_0}\bar{\nabla}\times\bar{E}$$
$$= \frac{j}{\omega\mu_0}\left[\bar{a}_x j0.1\beta\sin(10\pi x) + \bar{a}_z 0.1(10\pi)\cos(10\pi x)\right]e^{-j\beta z} \qquad (2)$$
$$\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H}$$
$$= \bar{a}_y \frac{0.1}{\omega^2\mu_0\epsilon_0}\left[(10\pi)^2+\beta^2\right]\sin(10\pi x)\, e^{-j\beta z} \qquad (3)$$

Equating (1) and (3): $(10\pi)^2+\beta^2 = \omega^2\mu_0\epsilon_0 = 400\pi^2$.

$$\longrightarrow \beta = \sqrt{300}\,\pi = 54.4\ \text{(rad/m)}.$$

From (2): $\bar{H}(x,z;t) = \mathcal{R}e\left(\bar{H}e^{j\omega t}\right)$
$$= -\bar{a}_x\, 2.30\times10^{-4}\sin(10\pi x)\cos(6\pi10^9 t - 54.4z)$$
$$- \bar{a}_z\, 1.33\times10^{-4}\cos(10\pi x)\sin(6\pi10^9 t - 54.4z)\ \text{(A/m)}$$

P. 6-21. $\bar{H}(x,z;t) = \bar{a}_y 2\cos(15\pi x)\sin(6\pi 10^9 t - \beta z)$ (A/m).

Phasor with sine reference:

$$\bar{H} = \bar{a}_y 2\cos(15\pi x)\cdot e^{-j\beta z} \qquad ①$$

$$\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H}$$

$$= \frac{1}{j\omega\epsilon_0}2\left[\bar{a}_x j\beta\cos(15\pi x)e^{-j\beta z} - \bar{a}_z 15\pi\sin(15\pi x)e^{-j\beta z}\right]. \qquad ②$$

$$\bar{H} = -\frac{1}{j\omega\mu_0}\bar{\nabla}\times\bar{E}$$

$$= \frac{2}{\omega^2\mu_0\epsilon_0}\left[\bar{a}_y\left(-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z}\right)\right]$$

$$= \bar{a}_y\frac{2}{\omega^2\mu_0\epsilon_0}\left[(15\pi)^2 + \beta^2\right]\cos(15\pi x)\cdot e^{-j\beta z} \qquad ③$$

Comparing ① and ③, we require

$$(15\pi)^2 + \beta^2 = \omega^2\mu_0\epsilon_0 = \frac{(6\pi 10^9)^2}{c^2}$$

$$= \frac{(6\pi 10^9)^2}{(3\times 10^8)^2} = 400\pi^2$$

$$\longrightarrow \beta = 13.2\pi = 41.6 \text{ (rad/m)}.$$

From ②, we have

$$\bar{E}(x,z;t) = \mathcal{I}m(\bar{E}e^{j\omega t})$$

$$= \bar{a}_x 496\cos(15\pi x)\sin(6\pi 10^9 t - 41.6z)$$

$$+ \bar{a}_z 565\sin(15\pi x)\cos(6\pi 10^9 t - 41.6z) \quad \text{(V/m)}.$$

Chapter 7

Plane Electromagnetic Waves

P. 7-1 a) In a source-free conducting medium with constitutive parameters ϵ, μ, and σ,

Eq. (7-62): $\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$

$$= \sigma E + \epsilon\frac{\partial\bar{E}}{\partial t}. \qquad ①$$

Eqs.(5-16a) & (7-61): $\bar{\nabla}\times\bar{\nabla}\times\bar{E} = \bar{\nabla}\bar{\nabla}\cdot\bar{E} - \bar{\nabla}^2\bar{E}$

$$= -\mu\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{H}). \qquad ②$$

Substituting ① in ② and noting that $\bar{\nabla}\cdot\bar{E}=0$, we obtain the wave equation in dissipative media:

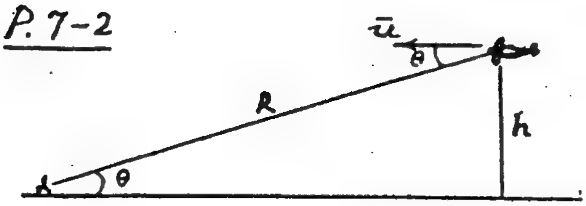
$$\bar{\nabla}^2\bar{E} - \mu\sigma\frac{\partial\bar{E}}{\partial t} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0. \qquad ③$$

Similarly for $\bar{H}$.

b) For time-harmonic fields: $\frac{\partial}{\partial t}\rightarrow(j\omega)$ and $\frac{\partial^2}{\partial t^2}\rightarrow(-\omega^2)$.
Wave equation ③ converts to Helmholtz's equation:

$$\bar{\nabla}^2\bar{E} - j\omega\mu\sigma\bar{E} + k^2\bar{E} = 0, \quad \text{where } k=\omega\sqrt{\mu\epsilon}.$$

P. 7-2



$$\Delta t = \frac{2R}{c} = 0.3\times10^{-3}\ (s).$$

$$R = \frac{\Delta t}{2}c = \frac{0.3\times10^{-3}}{2}\times3\times10^8$$
$$= 45\times10^3\ (m),$$
or 45 (km).

$$h = R\sin\theta = 45\times10^3\sin15.5° = 12\times10^3 (m), \text{ or } 12\ (km).$$

$$\Delta f = 2f\left(\frac{u}{c}\right)\cos15.5°.$$

$$\longrightarrow u = \frac{c\Delta f}{2f\cos15.5°} = 410.8\ (m/s), \text{ or about } 1.2 \text{ Mach}.$$

P. 7-3 Assume that $\bar{H}(\bar{R})$ has the form:

$$\bar{H}(\bar{R}) = \bar{H}_0 e^{-jk\bar{a}_k\cdot\bar{R}}.$$

Then, from Eq. (6-80b),

$$\bar{E}(\bar{R}) = \frac{1}{j\omega\epsilon}\nabla\times\bar{H}(\bar{R})$$

$$= \frac{1}{j\omega\epsilon}(-jk)\bar{a}_k\times\bar{H}(\bar{R})$$

$$= -\frac{1}{\omega\epsilon}(\omega\sqrt{\mu\epsilon})\,\bar{a}_k\times\bar{H}(\bar{R}),$$

or,

$$\bar{E}(\bar{R}) = -\eta\,\bar{a}_k\times\bar{H}(\bar{R}).$$

P. 7-4 $\bar{H} = \bar{a}_z\, 4\times10^{-6}\cos(10^7\pi t - k_0 y + \frac{\pi}{4})$ (A/m).

a) $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{10^7\pi}{3\times10^8} = \frac{\pi}{30} = 0.105$ (rad/m).

$\lambda = 2\pi/k_0 = 60$ (m).

At $t = 3\times10^{-3}$ (s), we require the argument of cosine in $\bar{H}$:

$$10^7\pi(3\times10^{-3}) - \frac{\pi}{30}y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}, \quad n = 0, 1, 2, \cdots$$

$$\longrightarrow y = \pm30n - 7.5\ (\text{m}) = 22.5 \pm n\lambda/2\ (\text{m}).$$

b) Use phasors with cosine reference:

$$\bar{H} = \bar{a}_z\, 4\times10^{-6}\, e^{j(-k_0 y + \pi/4)} \qquad (\text{A/m}).$$

From the result of Problem P. 7-3,

$$\bar{E} = -\eta_0\,\bar{a}_y\times\bar{a}_z\, 4\times10^{-6}\, e^{j(-k_0 y + \pi/4)}$$

$$= -\bar{a}_x\, 4\times10^{-6}\eta_0\, e^{j(-0.105y + \pi/4)}$$

$$= -\bar{a}_x\, 1.51\times10^{-3}\, e^{j(-0.105y + \pi/4)} \qquad (\text{V/m}).$$

The instantaneous expression for $\bar{E}$ is:

$$\bar{E}(y,t) = -\bar{a}_x\, 1.51\cos(10^7\pi t - 0.105y + \pi/4) \quad (\text{mV/m}).$$

P.7-5 Use phasors with cosine reference.

$$\bar{E}(z) = \bar{a}_x 2 e^{-jz/\sqrt{3}} + \bar{a}_y j e^{-jz/\sqrt{3}} \quad (\text{V/m}).$$

a) $\omega = 10^8$ (rad/s) $\longrightarrow$ $f = 10^8/2\pi = 1.59\times 10^7$ (Hz),
$\beta = 1/\sqrt{3}$ (rad/m) $\longrightarrow$ $\lambda = 2\pi/\beta = 2\sqrt{3}\,\pi$ (m).

b) $u = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = 3.$

c) Left-hand elliptically polarized.

d) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}} \quad (\Omega).$

$$\bar{H} = \frac{1}{\eta}\bar{a}_z \times \bar{E} = \frac{\sqrt{3}}{120\pi}\left(\bar{a}_y 2 e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}}\right),$$

$$\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x \sin(10^8 t - z/\sqrt{3}) + \bar{a}_y 2\cos(10^8 t - z/\sqrt{3})\right] \quad (\text{A/m}).$$

P.7-6 Let $\alpha = \omega t - kz$.

$$\bar{E} = \bar{a}_x E_{10}\sin\alpha + \bar{a}_y E_{20}\sin(\alpha+\psi)$$
$$= \bar{a}_x E_x + \bar{a}_y E_y .$$

$$\frac{E_x}{E_{10}} = \sin\alpha, \quad \frac{E_y}{E_{20}} = \sin(\alpha+\psi)$$
$$= \sin\alpha\cos\psi + \cos\alpha\sin\psi$$
$$= \frac{E_x}{E_{10}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi.$$

$$\left(\frac{E_y}{E_{20}} - \frac{E_x}{E_{10}}\cos\psi\right)^2 = \left[1-\left(\frac{E_x}{E_{10}}\right)^2\right]\sin^2\psi .$$

Rearranging:

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\,\frac{E_x E_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1,$$

which is the equation of an ellipse in E_x–E_y plane.

P.7-7 Given: $f = 3\times10^9$ (Hz), $\epsilon_r = 2.5$,

$$\tan\delta_c = \frac{\epsilon''}{\epsilon'} = 0.05.$$

a) Eq.(7-47): $\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c} = 2.48$ (Np/m).

$$e^{-\alpha x} = \frac{1}{2} \longrightarrow x = \frac{1}{\alpha}\ln 2 = 0.279 \text{ (m)}.$$

b) Eq. (7-49): $\eta_c = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = 238\angle 1.43^\circ$ (Ω)
$= 238\angle 0.008\pi$ (Ω).

Eq. (7-48): $\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 31.6\pi$ (rad/m).

$$\lambda = \frac{2\pi}{\beta} = 0.063 \text{ (m)}.$$

$$u_p = \frac{\omega}{\beta} = 1.897\times10^8 \text{ (m/s)}.$$

$$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 1.898\times10^8 \text{ (m/s)}.$$

c) At $x=0$, $\bar{E} = \bar{a}_y e^{j\pi/3}$, $\bar{H} = \frac{1}{\eta_c}\bar{a}_x\times\bar{E} = \bar{a}_z 0.210\, e^{j(\frac{\pi}{3} - 0.008\pi)}$

$$\bar{H}(x,t) = \bar{a}_z\, 0.210\, e^{-2.48x}\sin(6\pi10^9 t - 31.6\pi x + 0.325\pi) \text{ (A/m)}.$$

P.7-8 Both copper and brass are good conductors,

$\left(\frac{\sigma}{\omega\epsilon}\right)^2 \gg 1$: $\alpha = \sqrt{\pi f\mu\sigma}$, $\delta = \frac{1}{\alpha}$, $\eta_c = (1+j)\frac{\alpha}{\sigma}$.

a) $f = 1$ (MHz).

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$2.61(1+j)\times10^{-4}$	1.51×10^4	1.31×10^5	6.61×10^{-5}
Brass	$4.98(1+j)\times10^{-4}$	0.79×10^4	0.69×10^5	12.6×10^{-5}

b) $f = 1$ (GHz).

	η_c (Ω)	α (Np/m)	α (dB/m)	δ (m)
Copper	$8.25(1+j)\times10^{-3}$	4.79×10^5	4.16×10^6	2.09×10^{-6}
Brass	$1.58(1+j)\times10^{-3}$	2.51×10^5	2.18×10^6	3.99×10^{-6}

P.7-9 a) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \longrightarrow \sigma = \frac{1}{\pi f \mu \delta^2} = 0.99 \times 10^5$ (S/m).

b) At $f = 10^9$ (Hz), $\alpha = \sqrt{\pi f \mu \sigma} = 1.98 \times 10^4$ (Np/m).

$20 \log_{10} e^{-\alpha z} = -30$ (dB). $\longrightarrow z = \frac{1.5}{\alpha \log_{10} e} = 1.75 \times 10^{-4}$ (m) $= 0.175$ (mm).

P.7-10 $\mathcal{P}_{av} = |E|^2/2\eta_0 = 10^{-2}$ (W/cm²).

a) $|E| = \sqrt{0.02\eta_0} = 2.75$ (V/cm) $= 275$ (V/m),
$|H| = |E|/\eta_0 = 7.28 \times 10^{-3}$ (A/cm) $= 0.728$ (A/m).

b) $\mathcal{P}_{av} = |E|^2/2\eta_0 = 1300$ (W/m²).
$|E| = 990$ (V/m), $|H| = 2.63$ (A/m).

P.7-11 Assume circularly polarized plane wave:

$$\bar{E}(z,t) = \bar{a}_x E_0 \cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi),$$
$$\bar{H}(z,t) = \bar{a}_y \frac{E_0}{\eta} \cos(\omega t - kz + \phi) - \bar{a}_x \frac{E_0}{\eta} \sin(\omega t - kz + \phi).$$

Poynting vector, $\bar{\mathcal{P}} = \bar{E} \times \bar{H} = \bar{a}_z \frac{E_0^2}{\eta}\left[\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)\right]$
$= \bar{a}_z \frac{E_0^2}{\eta}$, a constant independent of t and z.

P.7-12 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$,

$$\bar{H} = \frac{1}{\eta} \bar{a}_R \times \bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi).$$
$$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E} \times \bar{H}^*) = \bar{a}_z \frac{1}{2\eta}(|E_\theta|^2 + |E_\phi|^2).$$

P.7-13 From Gauss's law: $\bar{E} = \bar{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \bar{E} \cdot d\bar{r} = \frac{\rho}{2\pi\epsilon} \ln\left(\frac{b}{a}\right). \longrightarrow \bar{E} = \bar{a}_r \frac{V_0}{r \ln(b/a)}.$$

From Ampère's circuital law, $\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}$.

Poynting vector, $\bar{\mathcal{P}} = \bar{E} \times \bar{H} = \bar{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}$.

Power transmitted over cross-sectional area:

$$P = \int_S \bar{\mathcal{P}} \cdot d\bar{s} = \frac{V_0 I}{2\pi \ln(b/a)} \int_0^{2\pi}\int_a^b \left(\frac{1}{r^2}\right) r\,dr\,d\phi = V_0 I.$$

P.7-14 $\bar{E}_i(x,t) = \bar{a}_y 50 \sin(10^8 t - \beta x)$ (V/m).

Use phasors with a sine reference.

$$\bar{E}_i(x) = \bar{a}_y 50 e^{-j\beta_1 x}$$

For air (medium 1): $\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times 10^8} = \frac{1}{3}$ (rad/m).

$\eta_1 = \eta_0 = 120\pi$ (Ω)

For lossless medium 2: $\beta_2 = \omega\sqrt{\mu_2\epsilon_2} = \frac{\omega}{c}\sqrt{\mu_{r_2}\epsilon_{r_2}} = \frac{4}{3}$ (rad/m).

$\eta_2 = \sqrt{\frac{\mu_0\mu_{r_2}}{\epsilon_0\epsilon_{r_2}}} = 2\eta_0 = 240\pi$ (Ω).

Eq.(7-25): $\bar{H}_i(x) = \frac{1}{\eta_0}\bar{a}_x \times \bar{E}_i = \bar{a}_z \frac{1}{\eta_0} 50 e^{-jx/3} = \bar{a}_z \frac{1}{2.4\pi} e^{-jx/3}$.

a)
$$\frac{E_{r0}}{E_{i0}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2-1}{2+1} = \frac{1}{3}.$$

$\bar{E}_r = \bar{a}_y \frac{50}{3} e^{jx/3} \longrightarrow \bar{E}_r(x,t) = \bar{a}_y \frac{50}{3}\sin(10^8 t + x/3)$ (V/m).

$\bar{H}_r = \frac{1}{\eta_0}(-\bar{a}_x)\times\bar{E}_r \longrightarrow \bar{H}_r(x,t) = -\bar{a}_z \frac{1}{7.2\pi}\sin(10^8 t + x/3)$ (A/m).

b) $\Gamma = \frac{1}{3}$, $\tau = 1 + \Gamma = \frac{4}{3}$.

$$S = \frac{1+\Gamma}{1-\Gamma} = 2.$$

c) $\bar{E}_t = (\tau E_{i0})\bar{a}_y e^{-j\beta_2 x} \longrightarrow \bar{E}_t(x,t) = \bar{a}_y \frac{200}{3}\sin(10^8 t - 4x/3)$ (V/m).

$\bar{H}_t = \frac{1}{\eta_2}\bar{a}_x \times \bar{E}_t$. $\longrightarrow \bar{H}_t(x,t) = \bar{a}_z \frac{1}{3.6\pi}\sin(10^8 t - 4x/3)$ (A/m).

P.7-15 $\frac{\sigma}{\omega\epsilon} = \frac{4}{10^8 \times 72 \times \frac{1}{36\pi}\times 10^{-9}} = 20\pi \gg 1$ (Good conductor).

a) Skin depth $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = 0.063$ (m) $= 6.3$ (cm) $= \frac{1}{\alpha} = \frac{1}{15.85}$.

$\eta_c = (1+j)\frac{\alpha}{\sigma} = 3.96(1+j) = 5.60 e^{j\pi/4}$ (Ω).

b) $\bar{H}(z,t) = \bar{a}_y 0.3 e^{-15.85z}\cos(10^8 t - 15.85z)$ (A/m).

$\bar{E}(z) = -\eta_c \bar{a}_z \times \bar{H}(z) \longrightarrow \bar{E}(z,t) = \bar{a}_x 1.68 e^{-15.85z}\cos(10^8 t - 15.85z + \frac{\pi}{4})$ (V/m).

c) $\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_z \frac{1.68\times 0.3}{2} e^{-31.7z}\cos\frac{\pi}{4} = \bar{a}_z 0.178 e^{-31.7z}$ (W/m²).

P.7-16 a) $\Gamma = \dfrac{E_{ro}}{E_{io}} = \dfrac{-\eta_1 H_{ro}}{\eta_1 H_{io}} = \dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}$.

$\longrightarrow \dfrac{H_{ro}}{H_{io}} = -\Gamma = \dfrac{\eta_1-\eta_2}{\eta_1+\eta_2}$.

b) $\tau = \dfrac{E_{to}}{E_{io}} = \dfrac{\eta_2 H_{to}}{\eta_1 H_{io}} = \dfrac{2\eta_2}{\eta_2+\eta_1}$.

$\longrightarrow \dfrac{H_{to}}{H_{io}} = \dfrac{\eta_1}{\eta_2}\tau = \dfrac{2\eta_1}{\eta_2+\eta_1}$.

P.7-17 Given $\bar{E}_i = E_0(\bar{a}_x - j\bar{a}_y)e^{-j\beta z}$.

a) Assume reflected $\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry})e^{j\beta z}$.

Boundary condition at $z=0$: $\bar{E}_i(0) + \bar{E}_r(0) = 0$.

$\longrightarrow \bar{E}_r(z) = E_0(-\bar{a}_x + j\bar{a}_y)e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

b) $\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s. \longrightarrow -\bar{a}_z \times [\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$. ($\bar{H}_2 = 0$ in perfect conductor.)

$\bar{H}_i(0) = \dfrac{1}{\eta_0}\bar{a}_z \times \bar{E}_i(0) = \dfrac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$, $\bar{H}_r(0) = \dfrac{1}{\eta_0}(-\bar{a}_z)\times\bar{E}_r(0) = \dfrac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$.

$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \dfrac{2E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y)$,

$\bar{J}_s = -\bar{a}_z \times \bar{H}_1(0) = \dfrac{2E_0}{\eta_0}(\bar{a}_x - j\bar{a}_y)$.

c) $\bar{E}_1(z,t) = \mathcal{R}e\,[\bar{E}_i(z) + \bar{E}_r(z)]e^{j\omega t}$

$= \mathcal{R}e\,E_0[(\bar{a}_x - j\bar{a}_y)e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y)e^{j\beta z}]e^{j\omega t}$

$= \mathcal{R}e\,E_0[-2j(\bar{a}_x - j\bar{a}_y)\sin\beta z]e^{j\omega t}$

$= 2E_0\sin\beta z\,(\bar{a}_x\sin\omega t - \bar{a}_y\cos\omega t)$.

P.7-18 For normal incidence:

$1 + \Gamma = \tau$, where $|\Gamma| \le 1$.

If $|\tau| = |\Gamma|$: $\Gamma < 0$, and $\eta_1 - \eta_2 = 2\eta_2$.

$\longrightarrow \eta_1 = 3\eta_2 \longrightarrow |\Gamma| = \frac{1}{2}$.

$S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = 3. \longrightarrow S_{dB} = 20\log_{10}3 = 9.54$ (dB).

P. 7-19 $\quad \bar{E}_i(z) = \bar{a}_x 10 e^{-j6z}$ (V/m).

$\longrightarrow \bar{H}_i(z) = \bar{a}_y \frac{10}{377} e^{-j6z}$ (A/m).

In air : $\beta_1 = 6 = \frac{\omega}{c} \longrightarrow \omega = 6c = 1.8\times 10^9$ (rad/s).

In lossy medium: $\epsilon_{r2} = 2.25$, $\tan\delta = \frac{\epsilon''}{\epsilon'} = 0.3$.

Eq. (7-47): $\alpha_2 = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_{r2}}}{c} = 1.35$ (Np/m).

Eq. (7-48): $\beta_2 = \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 9.10$ (rad/m).

Eq. (7-49): $\eta_2 = \sqrt{\frac{\mu'}{\epsilon'}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = \frac{377}{\sqrt{2.25}}\left(1 + j\frac{0.3}{2}\right) = 254 e^{j8.5°}$ (Ω).

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.208 e^{j159.9°}$

$\tau = 1 + \Gamma = 0.805 + j0.072 = 0.808 e^{j5.1°}$

a) $\bar{E}_r(z) = \bar{a}_x 2.08 e^{j(6z + 159.9°)}$ (V/m).

$\bar{H}_r(z) = \frac{1}{\eta_1}(-\bar{a}_z)\times\bar{E}_r(z) = \frac{1}{377}(-\bar{a}_z\times\bar{a}_x)\, 2.08 e^{j(6z+159.9°)}$

$= -\bar{a}_y 0.0055 e^{j(6z+159.9°)}$ (A/m).

$\bar{E}_t(z) = \bar{a}_x(\tau E_{i0}) e^{-\alpha_2 z} e^{-j\beta_2 z} = \bar{a}_x 8.08 e^{-1.35z} e^{-j(9.10z - 5.1°)}$ (V/m).

$\bar{H}_t(z) = \bar{a}_y \frac{8.08}{\eta_2} e^{-1.35z} e^{-j(9.10z - 5.1°)}$

$= \bar{a}_y 0.032 e^{-1.35z} e^{-j(9.10z + 3.4°)}$ (A/m).

b) $\quad S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.208}{1 - 0.208} = 1.53.$

c) $(\bar{\mathcal{P}}_{av})_1 = \frac{1}{2}\mathcal{R}e\left(\bar{E}_i \times \bar{H}_i^* + \bar{E}_r \times \bar{H}_r^*\right)$

$= \bar{a}_z\left(\frac{10^2}{2\times 120\pi} - \frac{2.08^2}{2\times 120\pi}\right) = \bar{a}_z 0.127$ (W/m²).

$(\bar{\mathcal{P}}_{av})_2 = \frac{1}{2}\mathcal{R}e\left(\bar{E}_t \times \bar{H}_t\right) = \bar{a}_z \frac{8.08^2}{2\times 254} e^{-2.70z}\cos 8.5°$

$= \bar{a}_z 0.127 e^{-2.70z}$ (W/m²).

P.7-20 Given $\bar{E}_i(x,z) = \bar{a}_y 10\, e^{-j(6x+8z)}$ (V/m).

a) $k_x = 6,\ k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m).

$\lambda = 2\pi/k = 2\pi/10 = 0.628$ (m); $f = c/\lambda = 4.78\times10^8$ (Hz); $\omega = kc = 3\times10^9$ (rad/s).

b) $\bar{E}_i(x,z;t) = \bar{a}_y 10\cos(3\times10^9 t - 6x - 8z)$ (V/m).

$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i$ $\quad\left(\bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8.\right)$

$= \frac{1}{120\pi}(\bar{a}_x 0.6 + \bar{a}_z 0.8)\times\bar{a}_y 10\, e^{-j(6x+8z)} = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x+8z)}$

$\bar{H}_i(x,z;t) = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)\cos(3\times10^9 t - 6x - 8z)$ (A/m).

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \left(\frac{\bar{k}}{k}\right)\cdot\bar{a}_z = 0.8 \longrightarrow \theta_i = \cos^{-1}0.8 = 36.9°$.

d) $\bar{E}_i(x,0) + \bar{E}_r(x,0) = 0 \longrightarrow \bar{E}_r(x,z) = -\bar{a}_y 10\, e^{-j(6x-8z)}$.

$\bar{H}_r(x,z) = \frac{1}{\eta_0}\bar{a}_{nr}\times\bar{E}_r(x,z)$ $\quad\left(\bar{a}_{nr} = \bar{a}_x 0.6 - \bar{a}_z 0.8.\right)$

$= -\left(\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x-8z)}$

e) $\bar{E}_1(x,z) = \bar{E}_i(x,z) + \bar{E}_r(x,z) = \bar{a}_y 10\left(e^{-j8z} - e^{j8z}\right)e^{-j6x}$

$= -\bar{a}_y j20\, e^{-j6x}\sin 8z$ (V/m).

$\bar{H}_1(x,z) = \bar{H}_i(x,z) + \bar{H}_r(x,z) = -\left(\bar{a}_x\frac{2}{15\pi}\cos 8z + \bar{a}_z\frac{j}{10\pi}\sin 8z\right)e^{-j6x}$ (A/m).

P.7-21 Snell's law of reflection: $\theta_r = \theta_i = 30°$.

Snell's law of refraction: $\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i = \frac{1}{3}$.

$\theta_t = 19.47°,\ \cos\theta_t = 0.943$.

$\eta_1 = \eta_0 = 377\ (\Omega)$, $\quad\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{377}{\sqrt{2.25}} = 251\ (\Omega)$.

a) $\Gamma_\perp = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.241$.

$\tau_\perp = 1 + \Gamma_\perp = 1 - 0.241 = 0.759$.

b) From Eq.(7-141): $\bar{E}_t(x,z) = \bar{a}_y\tau E_{i0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$.

$\beta_2 = \omega\sqrt{\mu_2\epsilon_2} = \pi$ (rad/m). $\longrightarrow \bar{E}_t(x,z;t) = \bar{a}_y 15.2\cos(2\pi10^8 t - 1.05x - 2.96z)$ (V/m).

From Eq.(7-142): $\bar{H}_t(x,z) = \frac{15.2}{251}(-\bar{a}_x\cos\theta_t + \bar{a}_z\sin\theta_t)\, e^{-j(1.05x+2.96z)}$

$\longrightarrow \bar{H}_t(x,z;t) = 0.06(-\bar{a}_x 0.943 + \bar{a}_z 0.333)\cos(2\pi10^8 t - 1.05x - 2.96z)$ (A/m).

P.7-22 From problem P.7-21:

$$\theta_r = \theta_i = 30°, \qquad \theta_t = 19.47°.$$

$$\eta_1 = 377\ (\Omega), \quad \eta_2 = 251\ (\Omega).$$

a) From Eq. (7-158): $\Gamma_{\parallel} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$.

$$\Gamma_{\parallel} = \frac{251\times 0.943 - 377\times 0.866}{251\times 0.943 + 377\times 0.866} = -0.159.$$

From Eq. (7-160):

$$\tau_{\parallel} = (1+\Gamma_{\parallel})\frac{\cos\theta_i}{\cos\theta_t} = 0.772.$$

b) $\bar{H}_i(x,z) = \bar{a}_y 0.053\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$.

$$\beta_1 = \frac{\omega}{c} = \frac{2\pi\times 10^8}{3\times 10^8} = \frac{2\pi}{3} \quad (\text{rad/m}).$$

$$\beta_2 = \omega\sqrt{\mu_2\epsilon_2} = \sqrt{\epsilon_{r2}}\,\beta_1 = \pi \quad (\text{rad/m}).$$

$$\bar{H}_i(x,z) = \bar{a}_y 0.053\, e^{-j(\pi x/3 + \pi z/\sqrt{3})} \qquad (\text{A/m}).$$

From Eq. (7-150): $\bar{E}_i(x,z) = 19.98(\bar{a}_x 0.866 - \bar{a}_z 0.5)e^{-j(\pi x/3 + \pi z/\sqrt{3})}$ (V/m).

$$E_{t0} = \tau_{\parallel} E_{i0} = 0.772\times 19.98 = 15.42\ (\text{V/m}).$$

From Eqs. (7-154) and (7-155):

$$\bar{E}_t(x,z) = 15.42(\bar{a}_x 0.943 - \bar{a}_z 0.333)e^{-j(1.05x + 2.96z)}.$$

$$\bar{H}_t(x,z) = \bar{a}_y 0.061\, e^{-j(1.05x + 2.96z)}.$$

Thus, with a cosine reference,

$$\bar{E}_t(x,z;t) = 15.42(\bar{a}_x 0.943 - \bar{a}_z 0.333)\cos(2\pi 10^8 t - 1.05x - 2.96z)\ (\text{V/m}).$$

$$\bar{H}_t(x,z;t) = \bar{a}_y 0.061 \cos(2\pi 10^8 t - 1.05x - 2.96z) \quad (\text{A/m}).$$

P. 7-24 Given $f = f_p/2$ and $\theta_i = 60°$.

$$\longrightarrow \eta_p = \eta_0/\sqrt{1-(f_p/f)^2} = -j\eta_0/\sqrt{3}, \quad \eta_p/\eta_0 = -j/\sqrt{3}.$$

From Eq.(7-118): $\sin\theta_t = \frac{\eta_p}{\eta_0}\sin\theta_i = -j/2$, $\cos\theta_t = \sqrt{5}/2$, $\cos\theta_i = 1/2$.

a) From Eq. (7-147): $\Gamma_\perp = \dfrac{(\eta_p/\eta_0)\cos\theta_i - \cos\theta_t}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = e^{j209°}$,

From Eq. (7-148): $\tau_\perp = \dfrac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = 0.5\,e^{-j75.5°}$,

b) From Eq. (7-158): $\Gamma_\parallel = \dfrac{(\eta_p/\eta_0)\cos\theta_t - \cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = e^{j76°}$,

From Eq. (7-159): $\tau_\parallel = \dfrac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = 0.177\,e^{-j38°}$.

$|\Gamma_\perp| = |\Gamma_\parallel| = 1$, but the phase shift of the reflected wave depends on the polarization of the incident wave. There are standing waves in the air and exponentially decaying transmitted waves in the ionosphere.

P. 7-25 $\quad k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2\mu_0\epsilon_2 - j\omega\mu_0\sigma_2.$ $\qquad$ ①

Continuity conditions at $z=0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega\sqrt{\mu_0\epsilon_0}\sin\theta_i = \beta_x = 2.09\times10^{-4}. \qquad ②$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z}. \qquad ③$$

Combining ①, ② and ③, we can solve for α_{2z} and β_{2z} in terms of ω, μ_0, ϵ_2, σ_2, and β_x. But, since

$$\beta_x^2 << \omega^2\mu_0\epsilon_2,$$

we have $\alpha_2 = \alpha_{2z} \cong \beta_{2z} \cong \frac{1}{\delta} = \sqrt{\pi f\mu_0\sigma_2} = 0.3974\ (m^{-1})$.

a) $\theta_t = \tan^{-1}\dfrac{\beta_x}{\beta_{2z}} \cong \tan^{-1}\dfrac{2.09}{0.3974}\times10^{-4} \cong 5.26\times10^{-4}$ (rad) $= 0.03°$.

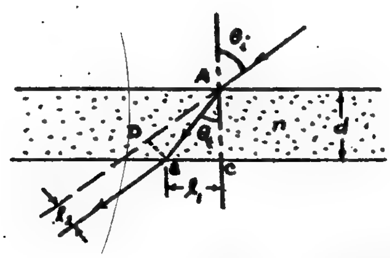
b) $\Gamma_{\perp} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_0\cos\theta_i}$ $\qquad \eta_2 = \dfrac{\alpha_2}{\sigma_2}(1+j) = 0.0993(1+j).$

$= \dfrac{2\times 0.0993(1+j)}{0.0993(1+j)+377\cos 88^\circ}$ $\qquad \cos\theta_t = \cos 0.03^\circ \cong 1.$

$\cong 0.0151(1+j) = 0.0214\, e^{j\pi/4}.$

c) $20\log_{10} e^{-\alpha_2 z} = -30. \longrightarrow z = \dfrac{1.5}{\alpha_2 \log_{10} e} = 8.69\ (m).$

P.7-26



a) Snell's law:

$\dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{1}{n},$

$\theta_t = \sin^{-1}\left(\dfrac{1}{n}\sin\theta_i\right).$

b) $\cos\theta_t = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}.$

$\ell_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\,\dfrac{\sin\theta_t}{\cos\theta_t} = \dfrac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}.$

c) $\ell_2 = \overline{BD} = \overline{AC}\,\sin(\theta_i-\theta_t) = \dfrac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$

$= d\sin\theta_i\left[1-\dfrac{\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right].$

P.7-27

a) $\sin\theta_c = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}. \longrightarrow \sin\theta_t = \sqrt{\dfrac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1$ for $\theta_i > \theta_c.$

$\cos\theta_t = -j\sqrt{\left(\dfrac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}.$

From Eqs. (7-141) and (7-142):

$\bar{E}_t(x,z) = \bar{a}_y E_{t0}\, e^{-\alpha_2 z} e^{-j\beta_{2x}x},$

$\bar{H}_t(x,z) = \dfrac{E_{t0}}{\eta_2}\left(\bar{a}_x j\alpha_2 + \bar{a}_z\sqrt{\dfrac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right)e^{-\alpha_2 z} e^{-j\beta_{2x}x},$

where $\beta_{2x} = \beta_2\sin\theta_t = \beta_2\sqrt{\dfrac{\epsilon_1}{\epsilon_2}}\sin\theta_i,$

$\alpha_2 = \beta_2\sqrt{\left(\dfrac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1},$

$E_{t0} = \dfrac{2\eta_2\cos\theta_i\, E_{i0}}{\eta_2\cos\theta_i - j\eta_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}}$ from Eq. (7-148).

b) $(\mathcal{P}_{av})_{2z} = \dfrac{1}{2}\mathcal{R}e\,(E_{ty}H_{tx}^*) = 0.$

P. 7-28 Given $\theta_i = \theta_c \longrightarrow \theta_t = \pi/2$, $\cos\theta_t = 0$.

a) From Eq. (7-148): $(E_{to}/E_{io})_\perp = 2$.

b) From Eq. (7-159): $(E_{to}/E_{io})_\parallel = 2\eta_2/\eta_1$.

c) $\bar{E}_i(x,z;t) = \bar{a}_y E_{i0} \cos\omega\left[t - \frac{n_1}{c}(x\sin\theta_i + z\cos\theta_i)\right]$,

$$\bar{E}_t(x,z;t) = \bar{a}_y 2E_{i0} e^{-\alpha z}\cos\omega(t - \tfrac{n_2}{c}x\sin\theta_t)$$
$$= \bar{a}_y 2E_{i0} e^{-\alpha z}\cos\omega(t - \tfrac{n_1}{c}x\sin\theta_i),$$

where $\alpha = \frac{n_2\omega}{c}\sqrt{(\frac{n_1}{n_2}\sin\theta_i)^2 - 1} = 0$, when $\theta = \theta_c$.

P. 7-29 a) $\theta_c = \sin^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}} = \sin^{-1}\sqrt{1/81} = 6.38°$.

b) $\theta_i = 20° > \theta_c$. $\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i = 3.08$. $\cos\theta_t = -j2.91$.

$$\Gamma_\perp = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = e^{j38°} = e^{j0.66}$$

c) $$\tau_\perp = \frac{2\sqrt{\epsilon_{r1}}\cos\theta_i}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = 1.89\,e^{j19°} = 1.89\,e^{j0.33}$$

d) The transmitted wave in air varies as $e^{-\alpha_2 z}e^{-j\beta_{2x}x}$.
where $\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = \frac{2\pi}{\lambda_0}(2.91)$.

Attenuation in air for each wavelength
$$= 20\log_{10} e^{-\alpha_2\lambda_0} = 159 \text{ (dB)}.$$

P. 7-30 When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0$, $\tau_1 = \frac{2\eta_2}{\eta_2 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_{t1}}{(\mathcal{P}_{av})_i} = \frac{\eta_0}{\eta_2}\tau_1^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45° > \theta_c = \sin^{-1}\left(\tfrac{1}{2}\right) = 30°.$$

On exit from the prism, $\tau_2 = \frac{2\eta_0}{\eta_2+\eta_0}$.

$$\frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_{t1}} = \frac{\eta_2}{\eta_0}\tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

$$\therefore \quad \frac{(\mathcal{P}_{av})_o}{(\mathcal{P}_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}\right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{(1+\sqrt{\epsilon_r})^2}\right]^2 = 0.79.$$

P.7-31 a) $n_0 \sin\theta_a = n_1 \sin(90° - \theta_c) = n_1 \cos\theta_c$

$$= n_1\sqrt{1-\sin^2\theta_c} = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}.$$

$$\sin\theta_a = \frac{1}{n_0}\sqrt{n_1^2-n_2^2} = \sqrt{n_1^2-n_2^2}. \qquad (n_0 = 1)$$

b) $N.A. = \sin\theta_a = \sqrt{2^2 - 1.74^2} = 0.9861,$

$$\theta_a = \sin^{-1} 0.9861 = 80.4°.$$

P. 7-32

a) For perpendicular polarization and $\mu_1 \neq \mu_2$:

$$\sin\theta_{B\perp} = \frac{1}{\sqrt{1+(\frac{\mu_1}{\mu_2})}}.$$

Under condition of no reflection:

$$\cos\theta_t = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\perp}} = \frac{1}{\sqrt{1+(\frac{\mu_1}{\mu_2})}}$$

$$= \sin\theta_{B\perp} \longrightarrow \theta_t + \theta_{B\perp} = \pi/2.$$

b) For parallel polarization and $\epsilon_1 \neq \epsilon_2$:

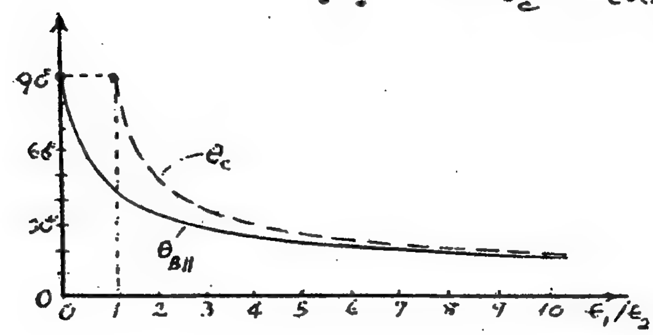
$$\sin\theta_{B\parallel} = \frac{1}{\sqrt{1+(\frac{\epsilon_1}{\epsilon_2})}}.$$

$$\cos\theta_t = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\parallel}} = \frac{1}{\sqrt{1+(\frac{\epsilon_1}{\epsilon_2})}}$$

$$= \sin\theta_{B\parallel} \longrightarrow \theta_t + \theta_{B\parallel} = \pi/2.$$

P.7-33 For two contiguous media with equal permeability and permittivities ϵ_1 and ϵ_2, we have from Eq. (7-120): $\theta_c = \sin^{-1}\sqrt{\epsilon_2/\epsilon_1}$, and from Eq. (7-164): $\theta_{B\parallel} = \tan^{-1}\sqrt{\epsilon_2/\epsilon_1}$.

$$\therefore \quad \sin\theta_c = \tan\theta_{B\parallel}.$$



ϵ_1/ϵ_2	θ_c	$\theta_{B\parallel}$
0	—	90°
0.5	—	54.7°
1	90°	45°
2	45°	35.3°
4	30°	26.6°
8	20.7°	19.5°
10	18.4°	17.6°

Chapter 8

Transmission Lines

P.8-1 Substituting Eqs. (8-17) and (8-18) in Eq. (8-43):

$$Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$$

a) $Z_0 = \frac{d'}{w}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow d' = \sqrt{2}\, d.$

b) $Z_0 = \frac{d}{w'}\sqrt{\frac{\mu}{2\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = \frac{1}{\sqrt{2}} w.$

c) $Z_0 = \frac{2d}{w'}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = 2w.$

d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} \longrightarrow u_{pa} = u_p/\sqrt{2}$ for part a.

$u_{pb} = u_p/\sqrt{2}$ for part b.

$u_{pc} = u_p$ for part c.

P.8-2 Given: $\sigma_c = 1.6\times10^7$ (S/m), $w = 0.02$ (m), $d = 2.5\times10^{-3}$ (m)

Lossy dielectric slab: $\mu = \mu_0$, $\epsilon_r = 3$, $\sigma = 10^{-3}$ (S/m).

$f = 5\times10^8$ (Hz).

a) $R = \frac{2}{w}\sqrt{\frac{\pi f \mu_0}{\sigma_c}} = 1.11$ (Ω/m).

$L = \mu\frac{d}{w} = 0.157$ (μH/m).

$G = \sigma\frac{w}{d} = 0.008$ (S/m).

$C = \epsilon\frac{w}{d} = 0.212$ (nF/m).

b) $\frac{|E_z|}{|E_y|} = \sqrt{\frac{\omega\epsilon}{\sigma_c}} = 4.167\times10^{-5}.$

c) $\omega L = 493.5 \gg R$, $\omega C = 0.667 \gg G$.

$\gamma \cong j\omega\sqrt{LC}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] = 0.129 + j18.14$ (m^{-1}),

$Z_0 \cong \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2j}\left(\frac{R}{\omega L} - \frac{G}{\omega C}\right)\right] = 27.21 + j0.13$ (Ω).

P. 8-3 a) For two-wire transmission line:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu}{\epsilon}}\cosh^{-1}\left(\frac{D}{2a}\right) = \frac{120}{\sqrt{\epsilon_r}}\ln\left[\frac{D}{2a}+\sqrt{\left(\frac{D}{2a}\right)^2-1}\right] = 300\ (\Omega).$$

$$\frac{D}{2a} = 21.27. \longrightarrow D = 25.5\times 10^{-3}\ (m).$$

b) For coaxial transmission line:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right) = \frac{60}{\sqrt{\epsilon_r}}\ln\left(\frac{b}{a}\right) = 75.$$

$$\frac{b}{a} = 6.52 \longrightarrow b = 3.91\times 10^{-3}\ (m).$$

P. 8-4 From Eq. (8-61): $\alpha = R\sqrt{\frac{C}{L}}$.

From Eqs. (8-28) and (8-29): $\alpha = \dfrac{R}{\frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\frac{b}{a}} = \dfrac{R}{75}$.

From Table 8-1: $R = \dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$.

For copper at 1 (MHz): $R_s = \sqrt{\dfrac{\pi f \mu_c}{\sigma_c}} = \sqrt{\dfrac{\pi 10^6\times 4\times 10^{-7}}{5.8\times 10^7}}$

$$= 2.61\times 10^{-4}\ (\Omega).$$

$$R = \frac{2.61\times 10^{-4}}{2\pi}\left(\frac{1}{0.6}+\frac{1}{3.91}\right)\times 10^3 = 0.08\ (\Omega).$$

$$\therefore\ \alpha = \frac{0.08}{75} = 1.065\times 10^{-3}\ (Np/m)$$
$$= 9.25\times 10^{-3}\ (dB/m).$$

P. 8-5 Eq. (8-38): $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$.

Given $Z_0 = 50 + j0\ (\Omega)$ — Purely real.

$\mathrm{Im}(Z_0) = 0 \longrightarrow \dfrac{R}{L} = \dfrac{G}{C} = k$ (Distortionless line).

Given: $\alpha = 0.01\ (dB/m) = 0.00115\ (Np/m)$.
$\beta = 0.8\pi\ (rad/m)$; $f = 10^8\ (Hz)$.

From Eqs. (8-48), (8-49) and (8-51): $\alpha = R\sqrt{\dfrac{C}{L}}$, $\beta = \omega\sqrt{LC}$, $Z_0 = \sqrt{\dfrac{L}{C}}$.

$R = \alpha Z_0 = 0.0576\ (\Omega/m)$, $L = \dfrac{\beta Z_0}{2\pi f} = 0.20\ (\mu H/m)$,

$G = \dfrac{RC}{L} = \dfrac{\alpha}{Z_0} = 23\ (\mu S/m)$, $C = \dfrac{L}{Z_0^2} = 80\ (pF/m)$.

P. 8-6 $(P_{av})_L = (P_{av})_i = \frac{1}{2}\mathcal{R}e[V_i I_i^*]$ $\qquad V_i = \frac{Z_i}{Z_g+Z_i}V_g$.

$$= \frac{|V_g|^2 R_i}{(R_g+R_i)^2+(X_g+X_i)^2}. \qquad I_i = \frac{V_g}{Z_g+Z_i}.$$

To maximize $(P_{av})_L$, set $\frac{\partial (P_{av})_L}{\partial R_i} = 0$, and $\frac{\partial (P_{av})_L}{\partial X_i} = 0$. $\Big\}$ $R_i = R_g$, $X_i = -X_g$ or $Z_i = Z_g^*$.

$$\text{Max.}\ (P_{av})_L = \frac{|V_g|^2}{4R_g} = (P_{av})_{Z_g}.$$

$\longrightarrow$ Max. power-transfer efficiency = 50%.

P. 8-7 $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$,

$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$.

At $z=0$; $V(0) = V_i = V_0^+ + V_0^-$, $\quad I(0) = I_i = I_0^+ + I_0^- = \frac{1}{Z_0}(V_0^+ - V_0^-)$.

$\longrightarrow V_0^+ = \frac{1}{2}(V_i + I_i Z_0)$, $\quad V_0^- = \frac{1}{2}(V_i - I_i Z_0)$.

a) $V(z) = \frac{1}{2}(V_i + I_i Z_0)e^{-\gamma z} + \frac{1}{2}(V_i - I_i Z_0)e^{\gamma z}$,

$I(z) = \frac{1}{2Z_0}(V_i + I_i Z_0)e^{-\gamma z} - \frac{1}{2Z_0}(V_i - I_i Z_0)e^{\gamma z}$.

b) $V(z) = V_i \cosh\gamma z - I_i Z_0 \sinh\gamma z$,

$I(z) = I_i \cosh\gamma z - \frac{V_i}{Z_0}\sinh\gamma z$.

P. 8-8 a) $-\frac{dV}{dz} = RI$, $\quad -\frac{dI}{dz} = GV$.

$$\begin{cases} \frac{d^2V}{dz^2} = RGV, \\ \frac{d^2I}{dz^2} = RGI. \end{cases}$$

b) $V(z) = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}$,

$I(z) = I_0^+ e^{-\alpha z} + I_0^- e^{\alpha z}$, $\quad \alpha = \sqrt{RG}$.

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = R_0 = \sqrt{\frac{R}{G}}.$$

We have $V(z) = \frac{1}{2}(V_i + I_i R_0)e^{-\alpha z} + \frac{1}{2}(V_i - I_i R_0)e^{\alpha z}$,

$$I(z) = \frac{1}{2}\left(\frac{V_i}{R_0} + I_i\right)e^{-\alpha z} - \frac{1}{2}\left(\frac{V_i}{R_0} - I_i\right)e^{\alpha z},$$

where $V_i = \frac{R_i}{R_g + R_i} V_g$ and $I_i = \frac{V_g}{R_g + R_i}$.

c) For an infinite line, $R_i = R_0$:

$$V(z) = \frac{R_0}{R_g + R_0} V_g e^{-\alpha z}, \qquad I(z) = \frac{V_g}{R_g + R_0} e^{-\alpha z}.$$

d) For a finite line of length ℓ terminated in R_L:

$$R_i = R_0 \frac{R_L + R_0 \tanh \alpha\ell}{R_0 + R_L \tanh \alpha\ell}.$$

P.8-9 Distortionless line: $R_0 = \sqrt{\frac{L}{C}} = 50\ (\Omega)$, $R = 0.5\ (\Omega/m)$,

$\tan\left(\frac{\sigma_d}{\omega\epsilon}\right) = \tan\left(\frac{G}{\omega C}\right) = 0.0018.$

$\longrightarrow \frac{G}{\omega C} \cong 0.0018;\ \frac{G}{C} = 8000\pi \times 0.0018 = 45.2 = \frac{R}{L}.$

$L = \frac{R}{G/C} = 0.011\ (H/m), \quad C = \frac{L}{R_0^2} = 4.42\ (\mu F/m).$

$\alpha = \frac{R}{R_0} = 0.010\ (Np/m), \quad \beta = \omega\sqrt{LC} = 5.55\ (rad/m).$

a) $V(z) = \frac{V_g R_0}{R_0 + R_g} e^{-\alpha z} e^{-j\beta z} = \frac{50}{9 + j3} e^{-0.01z} e^{-j5.55z};\ I(z) = \frac{V(z)}{50}.$

$\therefore V(z,t) = 5.27\, e^{-0.01z} \sin(8000\pi t - 5.55z - 0.322)\quad (V),$

$I(z,t) = 0.105\, e^{-0.01z} \sin(8000\pi t - 5.55z - 0.322)\quad (A).$

b) At $z = 50\ (m)$: $V(50,t) = 3.20 \sin(8000\pi t - 0.432\pi)\quad (V),$

$I(50,t) = 0.064 \sin(8000\pi t - 0.432\pi)\quad (A).$

c) $(P_{av})_L = \frac{1}{2}\mathcal{R}e\,|V_L I_L^*| = \frac{1}{2}(3.20 \times 0.064) = 0.102\ (W).$

P. 8-10 a) For a short-circuited line, set $Z_L=0$ in Eq.(8-78) to obtain:

$$Z_{is}=Z_0\tanh\gamma\ell=Z_0\frac{1-e^{-2\gamma\ell}}{1+e^{-2\gamma\ell}}.$$

For $\ell=\lambda/4$, $\beta\ell=\pi/2$, $\alpha\lambda/2\ll 1$.

$$Z_{is}=Z_0\frac{1-e^{-2\alpha(\lambda/4)}e^{-j\pi}}{1+e^{-2\alpha(\lambda/4)}e^{-j\pi}}\cong Z_0\frac{1+(1-\alpha\lambda/2)}{1-(1-\alpha\lambda/2)}$$
$$\cong 4Z_0/\alpha\lambda.$$

b) For an open-circuited line, set $Z_L\to\infty$ in Eq.(8-78) to obtain:

$$Z_{io}=Z_0\coth\gamma\ell=Z_0\frac{1+e^{-2\gamma\ell}}{1-e^{-2\gamma\ell}}.$$

For $\ell=\lambda/4$, $Z_{io}=Z_0\frac{1+e^{-(\alpha\lambda/2)}e^{-j\pi}}{1-e^{-(\alpha\lambda/2)}e^{-j\pi}}\cong Z_0\frac{1-(1-\alpha\lambda/2)}{1+(1-\alpha\lambda/2)}$

$$\cong Z_0\alpha\lambda/4.$$

P. 8-11 $\beta\ell=\frac{2\pi f}{c}\ell=\frac{8\pi}{3}=480°$,

$\tan\beta\ell=\tan 480°=-1.732$,

$$Z_i=Z_0\frac{Z_L+jZ_0\tan\beta\ell}{Z_0+jZ_L\tan\beta\ell}=50\frac{(40+j30)+j50(-1.732)}{50+j(40+j30)(-1.732)}$$
$$=26.3-j9.87\quad(\Omega).$$

P. 8-12 Given: $Z_{io}=Z_0\coth\gamma\ell=250\angle{-50°}\quad(\Omega)$,

$Z_{is}=Z_0\tan\gamma\ell=360\angle{20°}\quad(\Omega)$.

a) $Z_0=\sqrt{Z_{io}Z_{is}}=300\angle{-15°}=289.8-j77.6\ (\Omega)$.

$\tanh\gamma\ell=\sqrt{\frac{Z_{is}}{Z_{io}}}=1.2\angle{35°}=0.983+j688=\tanh(\alpha\ell+j\beta\ell)$.

$\ell=4\ (m)$, $\longrightarrow$ $\alpha=0.139\ (Np/m)$,

$\beta=0.235\ (rad/m)$.

b) $Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}$, $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$.

$\longrightarrow$ $R+j\omega L=\gamma Z_0$; $G+j\omega C=\frac{\gamma}{Z_0}$.

$\omega=\beta c=0.235\times 3\times 10^{8}=0.705\times 10^{8}\ (rad/m)$.

We obtain: $R=58.6\ (\Omega)$, $L=0.812\ (\mu H/m)$,

$G=0.246\ (mS/m)$, $C=12.4\ (pF/m)$.

P. 8-13 a) Since the line is very short compared to a wavelength, we may use Eqs. (8-81) and (8-83).

$$\therefore \quad \left.\begin{aligned} C &= \frac{54\times 10^{-12}}{0.6} = 9\times 10^{-11}\ (F/m), \\ L &= \frac{0.3\times 10^{-6}}{0.6} = 5\times 10^{-7}\ (H/m). \end{aligned}\right\} R_0 = \sqrt{\frac{L}{C}} = 74.5\ (\Omega).$$

$$\mu\epsilon = LC \longrightarrow \epsilon_r = \frac{LC}{\mu_0\epsilon_0} = 4.05.$$

b) $$\beta = \frac{\omega}{u_p} = 2\pi\times 10^7\sqrt{LC} = 0.42\ (rad/m); \quad \beta\ell = 0.42\times 0.6 = 0.252\ (rad) = 14.4^\circ$$

$$\therefore\ X_{io} = -R_0 \cot\beta\ell \doteq -\frac{1}{\omega C\ell} = -290\ (\Omega),$$

$$X_{is} = R_0 \tan\beta\ell \doteq \omega L\ell = 19.2\ (\Omega).$$

P. 8-14 For load impedance

$$Z_L = R_L + jX_L,$$

a) $$|\Gamma| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_0} - 1\right|}{\left|\frac{Z_L}{Z_0} + 1\right|} = \frac{\sqrt{(r_L-1)^2 + x_L^2}}{\sqrt{(r_L+1)^2 + x_L^2}},$$

where $r_L = R_L/Z_0$ and $x_L = X_L/Z_0$.

$$\longrightarrow x_L = \pm\left[\frac{\left(\frac{S-1}{S+1}\right)^2 (r_L+1)^2 - (r_L-1)^2}{1 - \left(\frac{S-1}{S+1}\right)^2}\right].$$

When $S=3$, $x_L = \pm\sqrt{(10r_L - 3r_L^2 - 3)/3}$.

b) $S = 3$ and $r_L = 150/75 = 2$.

$$\longrightarrow x_L = \pm\sqrt{5/3}.$$

$$X_L = x_L Z_0 = \pm 96.8\ (\Omega).$$

P. 8-15 For a lossless line, $Z_0 = R_0$.

$$|\Gamma|^2 = \left|\frac{(R_L - R_0) + jX_L}{(R_L + R_0) + jX_L}\right|^2 = \frac{(R_L - R_0)^2 + X_L^2}{(R_L + R_0)^2 + X_L^2}.$$

a) Set $\frac{\partial |\Gamma|^2}{\partial R_0} = 0. \longrightarrow R_0 = \sqrt{R_L^2 + X_L^2}$.

(A minimum S corresponds to a minimum $|\Gamma|$.)

For $Z_L = 40 + j30\ (\Omega)$, $R_0 = \sqrt{40^2 + 30^2} = 50\ (\Omega)$.

b) $$\text{Min. } |\Gamma| = \sqrt{\frac{R_0 - R_L}{R_0 + R_L}} = \sqrt{\frac{50-40}{50+40}} = \frac{1}{3}.$$

$$\text{Min. } S = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2.$$

P. 8-16 Lossless line of characteristic resistance R_0', length ℓ' and terminating in Z_L: (from Eq. 8-79)

$$Z_i = R_0' \frac{Z_L + jR_0' t}{R_0' + jZ_L t}, \quad t = \tan \beta\ell'.$$

$$\longrightarrow Z_L = R_0' \frac{Z_i - jR_0' t}{R_0' - jZ_i t}.$$

Now set $Z_i = 50\ (\Omega)$ and $Z_L = 40 + j10\ (\Omega)$.

$$40 + j10 = R_0' \frac{50 - jR_0' t}{R_0' - j50t}.$$

$$\longrightarrow \begin{cases} 40R_0' + 500t = 50R_0', \\ 10R_0' - 2000t = -(R_0')^2 t. \end{cases}$$

Solving: $R_0' = 38.7\ (\Omega)$,

and $t = \tan \beta\ell' = 0.775$.

$\longrightarrow \ell' = 0.105\lambda$.

P. 8-17 Eq. (8-79): $R_i + jX_i = R_0 \dfrac{R_L + jR_0 \tan\beta\ell}{R_0 + jR_L \tan\beta\ell}$.

Let $r_i = \dfrac{R_i}{R_0}$, $x_i = \dfrac{X_i}{R_0}$, $r_L = \dfrac{R_L}{R_0}$, and $t = \tan\beta\ell$.

$$r_i + jx_i = \frac{r_L + jt}{1 + jr_L t}.$$

$$\longrightarrow \begin{cases} r_L(1 + x_i t) = r_i, \\ t(1 - r_L r_i) = x_i. \end{cases}$$

Solving, we obtain:

$$r_L = \frac{1}{2r_i}\left\{(1 + r_i^2 + x_i^2) \pm \sqrt{(1 + r_i^2 + x_i^2)^2 - 4r_i^2}\right\},$$

$$t = \frac{1}{2x_i}\left\{-[1 - (r_i^2 + x_i^2)] \pm \sqrt{[1 - (r_i^2 + x_i^2)]^2 + 4x_i^2}\right\}.$$

$$\ell = \frac{\lambda}{2\pi}\tan^{-1} t.$$

P. 8-18 a) $|\Gamma| = \dfrac{S-1}{S+1} = \dfrac{2-1}{2+1} = \dfrac{1}{3}$.

To find θ_Γ, write Eq. (8-72) as

$$V(z') = \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}[1 + \Gamma e^{-j2\beta z'}]$$

$$= \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}[1 + |\Gamma| e^{j\theta_\Gamma} e^{-j2\beta z'}]$$

$$= \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}[1 + |\Gamma| e^{j\phi}], \quad \phi = \theta_\Gamma - 2\beta z'.$$

Voltage is minimum when $\phi = \pm\pi$,

or when $\theta_\Gamma = 2\left(\dfrac{2\pi}{\lambda}\right) \times 0.3\lambda - \pi = 0.2\pi$.

$$\therefore \quad \Gamma = \frac{1}{3}e^{j0.2\pi} = 0.270 + j0.196.$$

b) $Z_L = R_0\left(\dfrac{1+\Gamma}{1-\Gamma}\right) = 300\left(\dfrac{1.270 + j0.196}{0.730 - j0.196}\right)$

$= 466 + j206 \ (\Omega)$.

P. 8-19 Given: $V_g = 0.1\angle 0°$ (V), $Z_g = Z_0 = 50\ (\Omega)$,

$R_L = 25\ (\Omega) = 0.5 Z_0$. $\ell = \frac{\lambda}{8}$.

$\longrightarrow \Gamma_L = \frac{R_L - R_0}{R_L + R_0} = -\frac{1}{3}$.

a)

From Fig. 8-5,

$$V_i = \frac{Z_i}{Z_0 + Z_i} V_g, \qquad I_i = \frac{V_g}{Z_0 + Z_i},$$

where from Eq.(8-78),

$$Z_i = Z_0 \frac{0.5 Z_0 + j Z_0 \tan\beta\ell}{Z_0 + j0.5 Z_0 \tan\beta\ell} = Z_0 \frac{1 + j2\tan\beta\ell}{2 + j\tan\beta\ell}.$$

$$\therefore\ V_i = \frac{1 + j2\tan\beta\ell}{3(1 + j\tan\beta\ell)} V_g = \frac{1}{30}\left(\frac{1 + j2\tan\beta\ell}{1 + j\tan\beta\ell}\right) \quad \text{(V)}.$$

$$I_i = \frac{2 + j\tan\beta\ell}{3 Z_0 (1 + j\tan\beta\ell)} V_g = \frac{2}{3}\left(\frac{2 + j\tan\beta\ell}{1 + j\tan\beta\ell}\right) \quad \text{(mA)}.$$

For $\ell = \frac{\lambda}{8}$, $\beta\ell = \left(\frac{2\pi}{\lambda}\right)\frac{\lambda}{8} = \frac{\pi}{4}$, $\tan\beta\ell = 1$.

$$V_i = \frac{1}{30}\left(\frac{1+j2}{1+j1}\right) = 0.527\angle +18.4° \quad \text{(V)}.$$

$$I_i = \frac{2}{3}\left(\frac{2+j1}{1+j1}\right) = 1.054\angle -18.4° \quad \text{(mA)}.$$

When V_g is connected to the line, a voltage wave of an amplitude $\frac{Z_0}{Z_0 + Z_g} V_g$ travels toward the load R_L, arriving with an amplitude $V_L^+ = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-j\beta\ell}$, which causes a reflected wave with an amplitude $V_L^- = \Gamma_L V_L^+$.

The reflected wave travels back toward the generator and is not reflected there because $Z_g = Z_0$, and $\Gamma_g = 0$.

$$\therefore\ V_L = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-j\beta\ell}(1 + \Gamma_L) = \frac{1}{30} e^{-j\beta\ell} = 0.033\angle -45° \quad \text{(V)}.$$

Similarly, $I_L = \frac{V_g}{Z_0 + Z_g} e^{-j\beta\ell}(1 - \Gamma_L) = \frac{4}{3} e^{-j\beta\ell} = 1.333\angle -45°$ (mA).

b) $S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$.

c) $(P_{av})_L = \frac{1}{2}\mathcal{R}e(V_L I_L^*) = \frac{1}{2}\left(\frac{1}{30}\right)\left(\frac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5}$ (W) $= 0.022$ (mW)

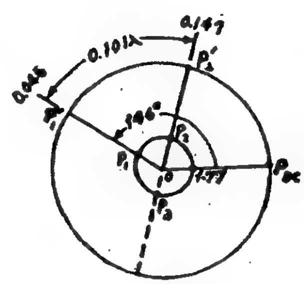
If $R_L = Z_0 = 50\ (\Omega)$, $\Gamma_L = 0$. $(P_{av})_L = \frac{V_g^2}{8 Z_0} = 0.025$ (mW).
(matched condition)

P. 8-20 $\quad f = 2\times10^{8}$ (Hz), $\quad \lambda = \frac{c}{f} = 1.5$ (m).

a) Open-circuited line, $\ell = 1$ (m), $\ell/\lambda = 0.667$.
Smith chart: Start from P_{oc} on the extreme right, rotate clockwise one complete revolution ($\Delta z' = \lambda/2$) and continue on for an additional 0.167λ to 0.417λ on the "wavelength toward generator" scale. Read $x = -j0.575$. $\longrightarrow$ $Z_i = 75\times(-j0.575) = -j43.1$ (Ω).
Draw a straight line from the $(0-j0.575)$ point through the center and intersect at $(0+j1.74)$ on the opposite side of the chart. $\longrightarrow$ $Y_i = \frac{1}{75}\times(j1.74) = j0.0232$ (S).

b) Short-circuited line, $\ell = 0.8$ (m), $\ell/\lambda = 0.533$.
Start from the extreme left point P_{sc}, rotate clockwise one complete revolution and continue on for an additional 0.033λ to read $x = j0.21 \longrightarrow Z_i = 75\times j0.21 = j15.8$ (Ω).
Draw a straight line from the $(0+j0.21)$ point through the center and intersect at $(0-j4.75)$ on the opposite side of the chart. $\longrightarrow$ $Y_i = \frac{1}{75}\times(-j4.75) = -j0.063$ (S).

P. 8-21



$z_L = \frac{1}{50}(30+j10) = 0.6+j0.2$.

a) 1. Locate $z_L = 0.6+j0.2$ on Smith chart (Point P_1).
2. With center at O draw a $|\Gamma|$-circle through P_1, intersecting OP_{oc} at 1.77. $\longrightarrow$ $S = 1.77$.

b) $\Gamma = \frac{1.77-1}{1.77+1}e^{j146°} = 0.28\,e^{j146°}$.

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.046 on "wavelengths toward generator" scale.
2. Move clockwise by 0.101λ to 0.147 (Point P_2').
3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.
4. Read $z_i = 1+j0.59$ at P_2.

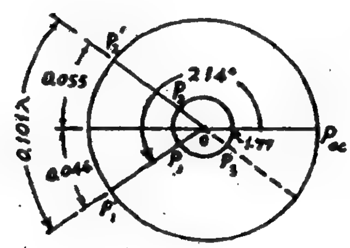
$$Z_i = 50z_i = 50+j29.5\ (\Omega).$$

d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 0.75 - j0.43$.

$$Y_i = \frac{1}{50} y_i = 0.015 - j0.009 \text{ (S)}.$$

e) There is no voltage minimum on the line, but $V_L < V_i$.

P. 8-22



$z_L = \frac{1}{50}(30 - j10) = 0.6 - j0.2$

a) Locate $z_L = 0.6 - j0.2$ on Smith chart (Point P_1). With center at O draw a $|\Gamma|$-circle through P_1, intersecting line OP_{oc} at 1.77. $\longrightarrow$ $S = 1.77$.

b) $\Gamma = 0.28\, e^{j214°}$

c) 1. Draw line OP_1, intersecting the periphery at P_1'. Read 0.454 on "wavelengths toward generator" scale.
 2. Move clockwise by 0.101λ to 0.055 (Point P_2').
 3. Join O and P_2', intersecting the $|\Gamma|$-circle at P_2.
 4. Read $z_i = 0.61 + j0.23$ at P_2.

$$Z_i = 50 z_i = 30.5 + j11.5 \ (\Omega).$$

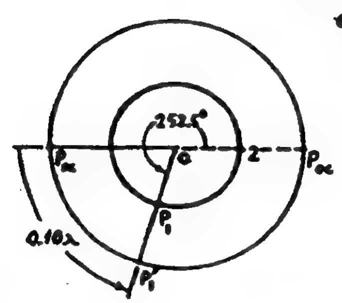
d) Extend line $P_2'P_2O$ to P_3. Read $y_i = 1.42 - j0.54$.

$$Y_i = \frac{1}{50} y_i = 0.0284 - j0.0108 \text{ (S)}.$$

e) There is a voltage minimum at $z_m' = 0.046\lambda$.

P. 8-23 $\quad \lambda/2 = 25, \quad \lambda = 50$ (cm).

First voltage minimum occurs at $z_m' = \frac{5}{50} = 0.1\lambda$.



a) 1. Start from P_{sc} and rotate counterclockwise 0.10λ toward the load to P_1'.
 2. Draw the $|\Gamma|$-circle, intersecting line OP_{oc} at 2 ($S = 2$).
 3. Join OP_1', intersecting the $|\Gamma|$-circle at P_1.

4. Read $z_L = 0.675 - j0.475$.

$\longrightarrow Z_L = 50 z_L = 33.75 - j23.75$ (Ω).

b) $\Gamma = \frac{2-1}{2+1} e^{j\theta_\Gamma} = \frac{1}{3} e^{j252.5°}$.

c) If $Z_L = 0$, the first voltage minimum would be at $z'_m = \lambda/2 = 25$ (cm) from the short-circuit.

P. 8-24 $\quad f = 2 \times 10^8$ (Hz).

$\lambda = 1.5$ (m). $\longrightarrow \ell = \frac{\lambda}{4} = 0.375$ (m).

Characteristic impedance of quarter-wire two-wire transmission line, $Z_0 = \sqrt{73 \times 300} = 148$ (Ω).

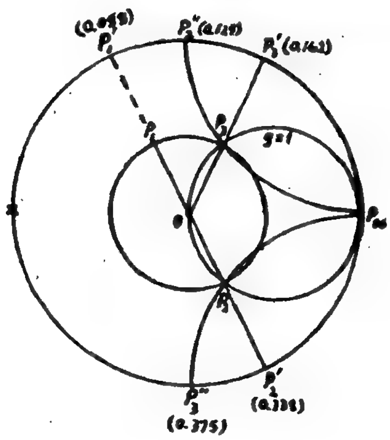
For a lossless air line, from Eqs. (8-23) and (8-24),

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\epsilon_0}} \cosh^{-1}\left(\frac{D}{2a}\right).$$

$$148 = 120 \cosh^{-1}\left(\frac{D}{2a}\right).$$

Given $D = 2$ (cm) $\longrightarrow a$ (wire radius) $= 0.54$ (cm).

P. 8-25 $\quad z_L = (25 + j25)/50 = 0.5 + j0.5, \quad y_L = 1 - j.$



a) See construction.

P_1: $z_L = 0.5 + j0.5$.

P_2: $y_L = 1 - j1 = y_{B1} \longrightarrow d_1 = 0$.

P_3: $y_{B2} = 1 + j1$.

$\longrightarrow d_2 = 0.162\lambda + (0.5 - 0.338)\lambda = 0.324\lambda$.

P_2'': $b_{B1} = j1 \longrightarrow \ell_1 = (0.25 + 0.125)\lambda = 0.375\lambda$.

P_3'': $b_{B2} = -j1 \longrightarrow \ell_2 = (0.375 - 0.25)\lambda = 0.125\lambda$.

P. 8-26 For $Z_0' = 75\,(\Omega) = 1.5\,Z_0$, $Y_0' = \frac{1}{1.5} Y_0 = 0.667\,Y_0$. } Compared to Problem P. 8-25.

The required normalized stub admittances are

$$b_{B1}' = -b_{B2}' = \frac{j}{0.667} = j1.5.$$

The locations of points P_2'' and P_3'' are now different.

We have: $\ell_1' = 0.406\lambda$ and $\ell_2' = 0.094\lambda$.

There are no changes in the locations of the stubs:

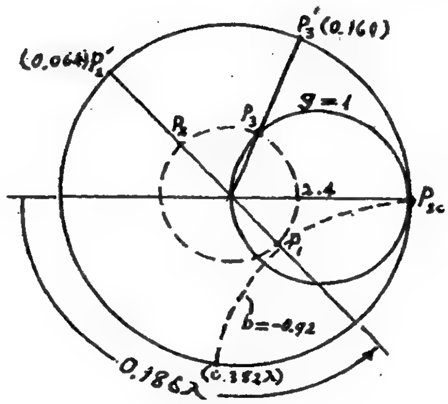
$$d_1' = d_1 = 0, \quad \text{and} \quad d_2' = d_2 = 0.324\lambda.$$

P. 8-27 Given: R_0 $75\,(\Omega)$, $S = 2.4$.

V_{min} at 0.335 (m) and 1.235 (m) from load.

$\longrightarrow \lambda = 2 \times (1.235 - 0.335) = 1.80$ (m).

First V_{min} at $\frac{0.335}{1.80} = 0.186\lambda$ from load.



Use a Smith chart.

1. Draw a centered circle (dashed) through $S=2.4$ point.
2. Locate point P_1 for z_L from V_{min} (point on extreme left) 0.186λ (clockwise) toward the load. $z_L = 1.39 - j0.98$.

a) $Z_L = 75\,z_L = 104.3 - j73.5\ (\Omega)$.

3. Locate the diametrically opposite point P_2 to find $y_L = 0.48 + j0.34$.

Read 0.064λ at point P_2'.

4. Use the Smith chart as an admittance chart and find the intersection of the $|\Gamma|$-circle with the $g=1$ circle at P_3: $y_B = 1 + j0.92$. Read 0.160λ at P_3'.

b) Location of stub $d = \underset{(\text{at } P_3')}{0.160\lambda} - \underset{(\text{at } P_2')}{0.064\lambda} = 0.096\lambda = 0.173$ (m).

Short-circuited stub length to give $b_B = -0.92$: $\ell = 0.382\lambda - 0.25\lambda = 0.132\lambda = 0.238$ (m).

P.8-28 Use Smith chart as an impedance chart. —

Same construction as that in problem P.8-25, except point P_{sc} would be at the extreme left (marked by a $\times$) and the $g=1$ circle becomes a $r=1$ circle.

$$P_1: \quad z_L = (25+j25)/50 = 0.5+j0.5.$$

Two possible solutions:

At P_3: $z_{s3} = 1+j1$.

$$\longrightarrow d_3 = (0.162\lambda - 0.088\lambda) = 0.074\lambda.$$

At P_2: $z_{s2} = 1-j1$.

$$\longrightarrow d_2 = (0.338\lambda - 0.088\lambda) = 0.250\lambda.$$

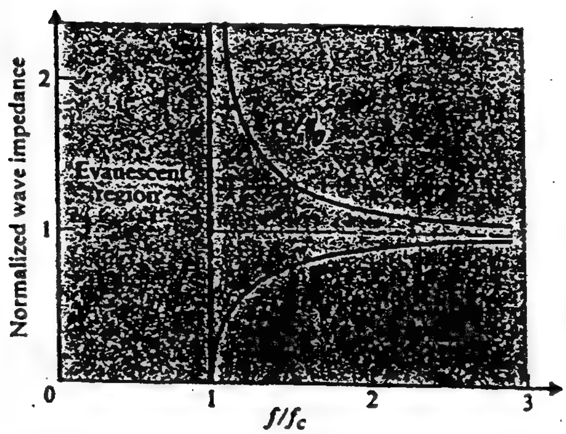
To achieve a match with a series stub having $R_0' = \frac{35}{50} R_0$, we need a normalized stub reactance $-j\frac{50}{35} = -j1.43$ for solution corresponding to P_3. From Smith chart we find the required stub length $\ell_3 = 0.347\lambda$.

Similarly for solution corresponding to P_2, a stub length with a normalized reactance $+j1.43$ is needed, which requires a stub length $\ell_2 = 0.153\lambda$.

Chapter 9

Waveguides and Resonators

P.9-1 We use Eqs. (9-34) and (9-39) for Z_{TM} and Z_{TE} respectively. For air, $\eta = \eta_0 = 120\pi\,(\Omega) = 377\,(\Omega)$.



a) The normalized wave impedances are plotted as shown.

b) $Z_{TM} = \eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$,

$Z_{TE} = \dfrac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$.

At $f = 1.1 f_c$, $\sqrt{1-\left(\frac{1}{1.1}\right)^2} = 0.417$.

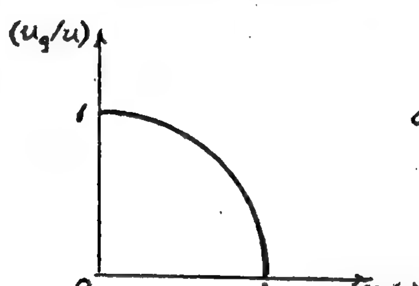
$Z_{TM} = 0.417\,\eta_0 = 157\,(\Omega)$.

$Z_{TE} = \dfrac{\eta_0}{0.417} = 904\,(\Omega)$.

At $f = 2.2 f_c$, $\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{1}{2.2}\right)^2} = 0.891$.

$Z_{TM} = 0.891\,\eta_0 = 336\,(\Omega)$, $Z_{TE} = \dfrac{\eta_0}{0.891} = 423\,(\Omega)$.

P.9-2 From Eq.(9-38), $\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{\omega}{u}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$.



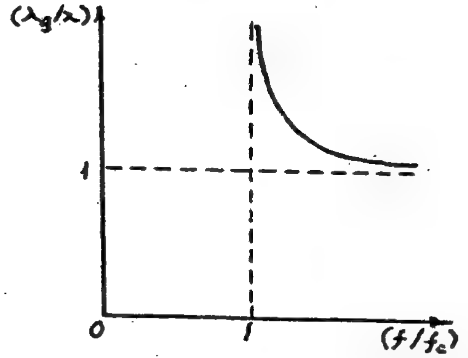
$u_p = \dfrac{\omega}{\beta} = \dfrac{u}{\sqrt{1-(f_c/f)^2}}$.

a) $u_g = \dfrac{1}{d\beta/d\omega} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$.

$\longrightarrow \left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$,

which indicates that the graph of (u_g/u) plotted versus (f_c/f) is a unit circle.

b) $\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi u}{\omega}\frac{1}{\sqrt{1-(f_c/f)^2}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}.$



$$\left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{1}{1-(f_c/f)^2}.$$

$$\longrightarrow \left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{(f/f_c)^2}{(f/f_c)^2-1}.$$

Graph shown on the left.

c) At $f/f_c = 1.25$,
$u_g/u = 0.60$, $\lambda_g/\lambda = 1.67$,
$u_p/u = 1.67$.

P.9-3 For TE waves between infinite parallel-plate waveguide in Fig. 9-3, we solve the following equation for $H_z^0(y)$: $\frac{d^2H_z^0(y)}{dy^2} + h^2H_z^0(y) = 0$,
with $H_z(y,z) = H_z^0(y)e^{-\gamma z}$. Boundary conditions to be satisfied at the conducting plates are:

$$\frac{dH_z^0(y)}{dy} = 0 \quad \text{at} \quad y=0 \text{ and } y=b.$$

a) Proper solution: $H_z^0(y) = B_n\cos\left(\frac{n\pi y}{b}\right)$; $h = \frac{n\pi}{b}$, $n=1,2,3,\cdots$

b) From Eq. (9-26): $f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}} = \frac{n}{2b\sqrt{\mu\epsilon}}.$

From TE_1 mode, $n=1$, $(f_c)_{TE_1} = \frac{u}{2b}.$

c) Instantaneous field expressions for TE_1 mode:

$H_z(y,z;t) = B_1\cos\left(\frac{\pi y}{b}\right)\cos(\omega t-\beta_1 z)$, $\beta_1 = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_{c1}}{f}\right)^2}$

$H_y(y,z;t) = -\frac{\beta_1 b}{\pi}B_1\sin\left(\frac{\pi y}{b}\right)\sin(\omega t-\beta_1 z).$

$E_x(y,z;t) = -\frac{\omega\mu b}{\pi}B_1\sin\left(\frac{\pi y}{b}\right)\sin(\omega t-\beta_1 z).$

P.9-4 Parts (a) and (b) similar to Problem P. 9-3

a) Phase expressions for field components of TE modes:

$$H_z(y,z) = B_n \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_n z},$$

$$\beta_n = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_{cn}}{f}\right)^2}, \quad n=1,2,3,\cdots$$

$$H_y(y,z) = \frac{j\beta_n b}{n\pi} B_n \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_n z}.$$

$$E_x(y,z) = \frac{j\omega\mu b}{n\pi} B_n \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_n z}.$$

b) From Eq. (9-26): $f_{cn} = \frac{n}{2b\sqrt{\mu\epsilon}}$.

c) Surface current densities: $\bar{J}_s = \bar{a}_n \times \bar{H}_t$.

On lower plate: $\bar{J}_{sl} = \bar{a}_y \times \bar{H}(0) = \bar{a}_x B_n e^{-j\beta_n z}$.

On upper plate: $\bar{J}_{su} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_x (-1)^{n+1} B_n e^{-j\beta_n z}$

$$= \begin{cases} \bar{J}_{sl} & \text{for } n \text{ odd}, \\ -\bar{J}_{sl} & \text{for } n \text{ even}. \end{cases}$$

P.9-5 a) $\lambda_g = 2 \times 2.65 = 5.30$ (cm).

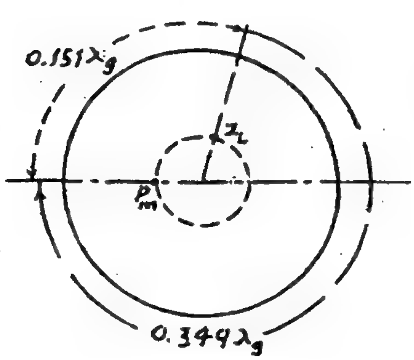
For TE_{10} mode: $f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 0.025} = 6\times 10^9$ (Hz).

$\lambda_c = 2a = 2\times 0.025 = 0.05$ (m).

From Eqs. (9-30) and (9-31): $\left(\frac{c}{\lambda_g}\right)^2 = f^2 - f_c^2$

$$\longrightarrow f = \sqrt{f_c^2 + \left(\frac{c}{\lambda_g}\right)^2} = \sqrt{6^2 + \left(\frac{0.3}{0.053}\right)^2} \times 10^9 = 8.25\times 10^9 \text{ (Hz)} = 8.25 \text{ (GHz)}.$$

b) Use Smith chart. Draw $|\Gamma| = \frac{2-1}{2+1} = \frac{1}{3}$ circle through $S=2$ point.



Shifting V_m toward load by $\frac{0.80}{5.30} = 0.151\lambda_g$ places point P_m from the load $(0.5-0.151)\lambda_g = 0.349\lambda_g$ toward the generator.

Read $z_L = 0.99 + j0.71$.

$$Z_{TE_{10}} = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}} = \frac{377}{\sqrt{1-(6/8.25)^2}} = 549 \ (\Omega)$$

$$\longrightarrow Z_L = (0.99 + j0.71)\times 549 = 544 + j390 \ (\Omega).$$

c) $P_{load} = 10\left(1 - \frac{1}{3^2}\right) = 8.89$ (W).

P.9-6 TM_{11} mode in air-filled rectangular waveguide operating at angular frequency $\omega = 2\pi f$ (see Eq. 9-65):

a) $$E_z^0(x,y) = E_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right).$$

Setting $H_z^0 = 0$ in Eqs. (9-11) through (9-14):

$$H_x^0(x,y) = \frac{j\omega\epsilon}{h_{11}^2}\frac{\partial E_z^0}{\partial y} = \frac{j\omega\epsilon}{h_{11}^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right),$$

$$H_y^0(x,y) = -\frac{j\omega\epsilon}{h_{11}^2}\frac{\partial E_z^0}{\partial x} = -\frac{j\omega\epsilon}{h_{11}^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right),$$

$$E_x^0(x,y) = -\frac{j\beta_{11}}{h_{11}^2}\frac{\partial E_z^0}{\partial x} = -\frac{j\beta_{11}}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right),$$

$$E_y^0(x,y) = -\frac{j\beta_{11}}{h_{11}^2}\frac{\partial E_z^0}{\partial y} = -\frac{j\beta_{11}}{h_{11}^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right),$$

where $h_{11}^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$, $\beta_{11} = \sqrt{\omega^2\mu\epsilon - h_{11}^2}$. Variations in z-direction are described by the factor $e^{-j\beta_{11}z}$.

b) From Eq. (9-26), $(f_c)_{TM_{11}} = \frac{h_{11}}{2\pi}c = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$.

$$(\lambda_c)_{TM_{11}} = \frac{c}{(f_c)_{TM_{11}}} = \frac{2}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{2ab}{\sqrt{a^2+b^2}}.$$

$$\lambda_g = \frac{2\pi}{\beta_{11}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{c}{\sqrt{f^2-f_c^2}}.$$

P.9-7 Rectangular waveguide: $a = 7.21$ (cm), $b = 3.40$ (cm).

Eq. (9-69): $$(\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}.$$

Modes with the shortest $\lambda_c < 5$ (cm) are:

Mode	TE_{10}	TE_{20}	TE_{01}	TE_{11}/TM_{11}
λ (cm)	14.4	7.20	6.80	6.15

a) For $\lambda = 10$ (cm), the only propagating mode is TE_{10}.

b) For $\lambda = 5$ (cm), the propagating modes are: TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11}.

P. 9-8 $\quad (f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{1}{2a\sqrt{\mu\epsilon}}F(m,n).$

a) $a=2b$, $F(m,n)=\sqrt{m^2+4n^2}$

Modes	$F(m,n)$
TE_{10}	1
TE_{01}, TE_{20}	2
TE_{11}, TM_{11}	$\sqrt{5}$
TE_{02}	4
TM_{12}	$\sqrt{17}$
TM_{22}	$\sqrt{20}$

b) $a=b$, $F(m,n)=\sqrt{m^2+n^2}$.

Modes	$F(m,n)$
TE_{10}, TE_{01}	1
TE_{11}, TM_{11}	$\sqrt{2}$
TE_{02}, TE_{20}	2
TM_{12}	$\sqrt{5}$
TM_{22}	$2\sqrt{2}$

P. 9-9 $\quad f = 3\times 10^9$ (Hz), $\lambda = c/f = 0.1$ (m).

Let $a = kb$, $1<k<2$. $\quad (f_c)_{mn} = \frac{3\times 10^8}{2a}\sqrt{m^2+k^2n^2}$.

a) $(f_c)_{10} = \frac{1.5\times 10^8}{a}$ for the dominant TE_{10} mode.

For $f > 1.2\,(f_c)_{10}$: $\quad a > 0.06$ (m).

The next higher-order mode is TE_{01} with $(f_c)_{01} = \frac{1.5\times 10^8}{b}$.

For $f < 0.8\,(f_c)_{01}$: $\quad b < 0.04$ (m).

We choose $a = 6.5$ (cm) and $b = 3.5$ (cm).

b) $u_p = \frac{c}{\sqrt{1-(\lambda/2a)^2}} = 4.70\times 10^8$ (m/s),

$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/2a)^2}} = 0.157$ (m) $= 15.7$ (cm),

$\beta = \frac{2\pi}{\lambda_g} = 40.1$ (rad/m),

$(Z_{TE})_{10} = \frac{\eta_0}{\sqrt{1-(\lambda/2a)^2}} = 590\ (\Omega)$.

P. 9-10 Given: $a=2.5\times10^{-2}$ (m), $b=1.5\times10^{-2}$(m), $f=7.5\times10^{9}$(Hz).

a) $\lambda=\frac{c}{f}=\frac{3\times10^{8}}{7.5\times10^{9}}=0.04$ (m).

$F_1=\sqrt{1-(\lambda/2a)^2}=0.60,$

$\lambda_g=\lambda/F_1=0.0667$ (m) $=6.67$ (cm),

$\beta=2\pi/\lambda_g=94.2$ (rad/m),

$u_p=c/F_1=5\times10^{8}$ (m/s),

$u_g=c\cdot F_1=1.8\times10^{8}$ (m/s),

$(Z_{TE})_{10}=\eta_0/F_1=200\pi=628$ (Ω).

b) $\lambda'=\frac{u}{f}=\frac{\lambda}{\sqrt{2}}=0.0283$ (m),

$F_2=\sqrt{1-(\lambda'/2a)^2}=0.825,$

$\lambda_g'=\lambda'/F_2=0.0343$ (m) $=3.43$ (cm),

$\beta'=2\pi/\lambda_g'=183.2$ (rad/m),

$u_p'=u/F_2=2.57\times10^{8}$ (m/s),

$u_g'=u\cdot F_2=1.75\times10^{8}$ (m/s),

$(Z_{TE})_{10}=\frac{\eta_0}{\sqrt{2}\,F_2}=323$ (Ω).

P. 9-11 Part (a) has been done in problem P. 9-6, part (a).

b) Use Eq. (7-79) to find the average power transmitted along the waveguide.

$$P_{av}=\frac{1}{2}\int_0^b\int_0^a\left[E_x^0H_y^0-E_y^0H_x^0\right]dx\,dy$$

$$=\frac{\omega\epsilon\beta_{11}E_0^2ab}{8\left[\left(\frac{\pi}{a}\right)^2-\left(\frac{\pi}{b}\right)^2\right]}.$$

P. 9-12 a) $E_z(x,y,z;t)=E_0\sin(100\pi x)\sin(100\pi y)\cos(2\pi10^{10}t-\beta z)$

$$=E_0\sin\left(\frac{2\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\cos(2\pi10^{10}t-\beta z).$$

Mode of operation: TM_{21}. $\omega=2\pi f=2\pi10^{10}$ (rad/s).

b) $(f_c)_{21} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{2}{0.05}\right)^2+\left(\frac{1}{0.025}\right)^2}$

$= 8.48\times10^9\ (Hz).$

$\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi10^{10}}{3\times10^8}\sqrt{1-\left(\frac{8.48}{10}\right)^2} = 111\ (rad/m).$

Eq. (9-34): $(Z_{TM})_{21} = \eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 377\sqrt{1-\left(\frac{8.48}{10}\right)^2}$

$= 377\times0.53 = 200\ (\Omega).$

$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^8}{10^{10}\times0.53} = 0.0566\ (m) = 5.66\ (cm).$

P.9-13 TE mode in 0.025 (m) × 0.025 (m) air-filled square waveguide:

$$H_z(x,y,z;t) = H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(\omega t-\beta z)$$
$$= 0.3\cos(80\pi y)\cos(\omega t-280z).$$

a) $\frac{n\pi}{b} = 80\pi = \frac{2\pi}{0.025} \longrightarrow n=2;\ m=0.$

$\longrightarrow TE_{02}$ mode.

b) From Eq. (9-68):

$(f_c)_{02} = \frac{c}{2}\frac{2}{b} = \frac{c}{b} = \frac{3\times10^8}{0.025} = 1.2\times10^{10}\ (Hz) = 12\ (GHz).$

From Eq. (9-38): $\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi}{c}\sqrt{f^2-f_c^2}.$

$f = \sqrt{\left(\frac{\beta c}{2\pi}\right)^2+f_c^2} = \sqrt{\left(\frac{280\times3\times10^8}{2\pi}\right)^2+(1.2\times10^{10})^2} = 1.8\times10^{10}\ (Hz)$
$= 18\ (GHz).$

$Z_{TE} = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}} = \frac{377}{\sqrt{1-(1.2/1.8)^2}} = 506\ (\Omega),$

$\lambda_g = \lambda/\sqrt{1-(f_c/f)^2} = c/f\sqrt{1-(f_c/f)^2} = 2.24\times10^{-2}\ (m) = 2.24\ (cm).$

c) $P_{av} = \frac{1}{2}\int_0^b\int_0^b \frac{|E_x|^2}{2Z_{TE}}\,dx\,dy = \frac{\omega^2\mu_0^2H_0^2}{4Z_{TE}}\int_0^b \sin^2\left(\frac{2\pi}{b}x\right)dx$

$= \frac{(2\pi f)^2\mu_0^2H_0^2}{4Z_{TE}}\left(\frac{b^2}{2}\right) = 280\ (W).$

P. 9-14 Substituting Eq. (9-97) in Eq. (9-24):

a) $$\gamma = j\left[\omega^2\mu\epsilon\left(1-j\frac{\sigma_d}{\omega\epsilon}\right)-h^2\right]^{1/2}$$
$$= j\sqrt{\omega^2\mu\epsilon-h^2}\left\{1-j\omega\mu\sigma_d(\omega^2\mu\epsilon-h^2)^{-1}\right\}^{1/2}$$
$$\cong j\sqrt{\omega^2\mu\epsilon-h^2}\left\{1-\frac{j\omega\mu\sigma_d}{2}(\omega^2\mu\epsilon-h^2)^{-1}\right\}.$$

From Eq. (9-28), $\sqrt{\omega^2\mu\epsilon-h^2} = \omega\sqrt{\mu\epsilon}\sqrt{1-(f_c/f)^2}$.

Hence, $$\gamma = \alpha_d + j\beta,$$

with $$\alpha_d = \frac{\sigma_d}{2}\sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1-(f_c/f)^2}} = \frac{\sigma_d\eta}{2\sqrt{1-(f_c/f)^2}}.$$

b)

At $f = 4\times10^9$ (Hz), TE_{10} is the only propagating mode which has a cutoff frequency of
$$(f_c)_{TE_{10}} = \frac{u}{2a} = \frac{c/\sqrt{\epsilon_r}}{2a} = \frac{3\times10^8/\sqrt{4}}{2\times0.025} = 3\times10^9\ \text{(Hz)}.$$

Thus, $$\alpha_d = \frac{3\times10^{-5}\times377}{2\sqrt{1-(3/4)^2}} = 0.0085\ \text{(Np/m)} = 0.074\ \text{(dB/m)}.$$

P. 9-15 $(f_c)_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$.

a) $(f_c)_{10} = \frac{c}{2a} = \frac{3\times10^8}{2\times0.025} = 6\times10^9\ \text{(Hz)} = 6\ \text{(GHz)}$.

Next higher mode:

	$b=0.25a$	$b=0.50a$	$b=0.75a$	
$(f_c)_{01} = \frac{c}{2b}$	24	12	8	(GHz)
$(f_c)_{11} = \frac{c}{2a}\sqrt{1+\left(\frac{a}{b}\right)^2}$	24.7	13.4	10	(GHz)
$(f_c)_{20} = \frac{c}{a}$	12	12	12	(GHz)
	0.85×12	0.85×12	0.85×8	
Usable bandwidth is from $1.15\times6 = 6.9$ (GHz) to:	10.2	10.2	6.8	(GHz)
Permissible bandwidth:	3.3 (GHz)	3.3 (GHz)	None <6.9	

b) From Eq. (9-101): $P_{av} = \frac{E_0^2ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 21.34\left(\frac{b}{a}\right)$.

$\longrightarrow$ $P_{av} = 5.3$ (W) for $b = 0.25a$, and 10.7 (W) for $b = 0.50a$.

P. 9-16 From Eq. (9-103): $f_{mnp} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$.

$a = 0.08\,(m)$, $b = 0.06\,(m)$, $d = 0.05\,(m)$.

$f_{mnp} = 1.5\times10^8 F(m,n,p)$, $F(m,n,p) = 100\sqrt{\left(\frac{m}{8}\right)^2 + \left(\frac{n}{6}\right)^2 + \left(\frac{p}{5}\right)^2}$.

Eight lowest-order modes and their resonant frequencies:

Modes	$F(m,n,p)$	f_{mnp} in (GHz)
TM_{110}	20.83	3.125
TE_{101}	23.58	3.538
TE_{011}	26.03	3.905
TE_{111}, TM_{111}	28.88	4.332
TM_{210}	30.05	4.507
TE_{201}	32.02	4.802
TM_{120}	35.60	5.340

P. 9-17 a) Since $d > a > b$, the lowest-order resonant mode is TE_{101} mode.

$$f_{101} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{d^2}} = 4.802\times10^9\,(Hz) = 4.802\ (GHz).$$

b) From Eq. (9-120):

$$Q_{101} = \frac{\pi f_{101}\mu_0 abd(a^2+d^2)}{R_s[2b(a^3+d^3)+ad(a^2+d^2)]} \qquad \left(R_s = \sqrt{\frac{\pi f_{101}\mu_0}{\sigma}}\right)$$

$$= \frac{\sqrt{\pi f_{101}\mu_0\sigma}\ abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)}$$

$$= 6{,}869.$$

From Eqs. (9-114) and (9-115):

$$W_e = \frac{1}{4}\epsilon_0\mu_0^2 a^3 b d f_{101}^2 H_0^2 = 0.0773\times10^{-12}\,(J) = 0.0773\,(pJ),$$

$$W_m = \frac{\mu_0}{16} abd\left(\frac{a^2}{d^2}+1\right)H_0^2 = 0.0773\,(pJ) = W_e.$$

P. 9-18 $\epsilon_r = 2.5$.

a) $f_{101} = \frac{u}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}} = \frac{1}{\sqrt{\epsilon_r}}(f_{101})_{\epsilon_0} = 3.037$ (GHz).

b) $Q_{101} = \frac{1}{(\epsilon_r)^{1/4}}(Q_{101})_{\epsilon_0} = 5{,}462$.

c) $W_e = (W_e)_{\epsilon_0} = 0.0773$ (pJ) $= W_m$.

P. 9-19 $\delta = \frac{1}{\sqrt{\pi f_{101}\mu_0\sigma}}$, $f_{101} = \frac{c}{2}\sqrt{\frac{2}{a^2}} = \frac{c}{\sqrt{2}\,a}$.

a) $Q_{101} = \frac{a}{3\delta} = \frac{a}{3}\sqrt{\pi c\mu_0\sigma/\sqrt{2}a} = 6{,}500$.

$19{,}500\,(2)^{1/4} = \sqrt{a}\sqrt{\pi\, 3\times10^8(4\pi10^{-7})(1.57\times10^7)}$

$\longrightarrow a = 0.0289$ (m) $= 2.89$ (cm).

b) $f_{101} = \frac{c}{\sqrt{2}\,a} = 7.34$ (GHz).

c) For copper, $\sigma = 5.80\times10^7$ (S/m).

$Q_{101} \propto \sqrt{\sigma}$.

$= 6{,}500\sqrt{\frac{5.80}{1.57}} = 12{,}493$.

P. 9-20 $Q_{101} = \frac{\sqrt{\pi f_{101}\mu_0\sigma}\; abd(a^2+d^2)}{2b(a^3+d^3)+ad(a^2+d^2)}$.

a) For $a = d = 1.8\,b = 0.036$ (m), $b = 0.02$ (m).

$f_{101} = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}} = 1.179\times10^8\left(\frac{1}{b}\right) = 5.89\times10^9$ (Hz)

$Q_{101} = 10.23\sqrt{\sigma b} = 11{,}018$.

b) For $Q'_{101} = 1.20\,Q_{101} \longrightarrow b' = 1.20^2 b = 1.44\times0.02$

$= 0.0288$ (m) $= 2.88$ (cm).

Chapter 10

Antennas and Antenna Arrays

P. 10-1 From Eqs. (10-12) and (10-14):

$$G_D(\theta,\phi) = \frac{4\pi R^2 \mathcal{P}_{av}}{P_r}.$$

Maximum G_D at $\mathcal{P}_{av}$ occur at $\theta = \pi/2$.

$$\mathcal{P}_{av} = \frac{DP_r}{4\pi R^2} = \frac{E_0^2}{2\eta_0}.$$

$$E_0^2 = \frac{\eta_0 D P_r}{2\pi R^2} \;; \quad D = 1.5, \; P_r = 0.70\times 15\times 10^3 \text{ (W)}.$$

$$\longrightarrow E_0 = 0.0972 \text{ (V/m)} = 97.2 \text{ (mV/m)}.$$

$$H_0 = \frac{E_0}{\eta_0} = 0.258 \text{ (mA/m)}.$$

P. 10-2 a) $D = \dfrac{U_{max}}{U_{av}}$. $\quad U_{max} = 50.$

$$U_{av} = \frac{1}{4\pi}\int U\, d\Omega$$

$$= \frac{50}{4\pi}\int_{-\pi/2}^{\pi/2}\int_0^{\pi}(\sin^2\theta\cos\phi)\sin\theta\, d\theta\, d\phi$$

$$= 2.65 \text{ (W/sr)}.$$

$$\longrightarrow D = \frac{50}{2.65} = 18.85, \quad \text{or } 12.75 \text{ (dB)}.$$

b) $U_{av} = \dfrac{P_r}{4\pi}$.

$$P_r = 4\pi U_{av} = 4\pi\times 2.65 = 33.3$$

$$= \frac{1}{2} I_i^2 R_r.$$

$$\longrightarrow R_r = \frac{2P_r}{I_i^2} = \frac{2\times 33.3}{2^2} = 16.7\ (\Omega).$$

P.10-3 Equation of continuity: $\bar{\nabla}\cdot\bar{J} = -j\omega\rho$

$$\longrightarrow \rho_\ell = \frac{j}{\omega}\frac{dI(z)}{dz}.$$

a) $I(z) = I_0 \cos 2\pi z \longrightarrow \rho_\ell = -\frac{I_0}{c}\sin 2\pi z.$

$\beta = \frac{2\pi}{\lambda} = 2\pi.$

$\longrightarrow$ Wavelength $\lambda = 1$ (m).

b) $I(z) = I_0\left(1 - \frac{4}{\lambda}|z|\right) \longrightarrow \rho_\ell = \begin{cases} -j\frac{2I_0}{\pi c} & \text{for } z>0, \\ +j\frac{2I_0}{\pi c} & \text{for } z<0. \end{cases}$

P.10-4 $\lambda = \frac{3\times10^8}{10^6} = 300$ (m), $\frac{d\ell}{\lambda} = \frac{15}{300} = \frac{1}{20} \ll 1$ (Hertzian dipole).

a) Radiation resistance, $R_r = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 = 1.97$ (Ω).

b) Eq. (10-30): $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = \sqrt{\frac{\pi\, 10^6 (4\pi\times10^{-7})}{5.80\times10^7}} = 2.61\times10^{-4}$ (Ω).

Eq. (10-29): $R_\ell = R_s\left(\frac{d\ell}{2\pi a}\right) = 0.031$ (Ω) $\longrightarrow \eta_r = \frac{R_r}{R_r+R_\ell} = 98.5\%.$

c) $\left.\begin{array}{l}\text{Eq. (10-24): } P_r = \frac{I^2(d\ell)^2}{12\pi}\eta_0\beta^2 \\ \text{Eq. (10-10): } |E_\theta|^2_{max.} = \left(\frac{I\,d\ell}{4\pi}\right)^2\frac{\eta_0^2\beta^2}{R^2}\end{array}\right\} \longrightarrow |E_\theta|_{max.} = \frac{1}{R}\sqrt{90P_r} = 19$ (mV/m).

P.10-5 $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = \sqrt{\frac{\pi(10)^8(4\pi 10^{-7})}{1.57\times10^7}} = 5.01\times10^{-3}$ (Ω).

$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^8} = 3$ (m).

Dipole length $= 1.5$ (m) $\longrightarrow \frac{\lambda}{2}$ dipole.

$\longrightarrow R_r = 73.1$ (Ω).

Power lost, $P_\ell = \frac{R_s}{2\pi a}\int_{-\lambda/4}^{\lambda/4}\frac{1}{2}(I_0\cos\beta z)^2 dz$

$$= \frac{R_s}{2\pi a}\left(\frac{I_0^2}{2}\right)\frac{1}{\beta}\int_{-\pi/2}^{\pi/2}\cos^2 x\, dx = 0.598\left(\frac{I_0^2}{2}\right).$$

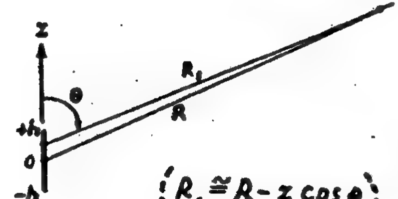
$$P_r = \left(\frac{I_0^2}{2}\right)R_r = 73.1\left(\frac{I_0^2}{2}\right).$$

$$\xi_r = \frac{P_r}{P_r+P_\ell} = \frac{73.1}{73.1+0.598} = 0.992, \text{ or } 99.2\%.$$

P.10-6 a) $E_\theta = j\dfrac{I_0\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\displaystyle\int_{-h}^{h}\left(1-\frac{|z|}{h}\right)e^{j\beta z\cos\theta}dz$

$$= j\frac{I_0\eta_0\beta\sin\theta}{2\pi R}e^{-j\beta R}\int_0^h\left(1-\frac{z}{h}\right)\cos(\beta z\cos\theta)dz$$

$$= \frac{j60I_0}{(\beta h)R}e^{-j\beta R}F(\theta),$$



$\{R_1 \cong R - z\cos\theta\}$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{jI_0}{(\beta h)2\pi R}e^{-j\beta R}F(\theta),$$

$$F(\theta) = \frac{\sin\theta[1-\cos(\beta h\cos\theta)]}{\cos^2\theta}.$$

In case $\beta h \ll 1$, $\cos(\beta h\cos\theta) \cong 1 - \frac{1}{2!}(\beta h\cos\theta)^2$, and

$$F(\theta) \cong \frac{1}{2}(\beta h)^2\sin\theta.$$

$$\therefore \quad E_\theta = \frac{j60I_0}{R}e^{-j\beta R}\left(\tfrac{1}{2}\beta h\sin\theta\right) = \frac{j30\beta h}{R}I_0e^{-j\beta R}\sin\theta,$$

$$H_\phi = \frac{jI_0}{2\pi R}e^{-j\beta R}\left(\tfrac{1}{2}\beta h\sin\theta\right) = \frac{j\beta h}{4\pi R}I_0e^{-j\beta R}\sin\theta.$$

b) $P_r = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}E_\theta H_\phi^* R^2\sin\theta\, d\theta\, d\phi = \frac{I_0^2}{2}\left[80\pi^2\left(\frac{h}{\lambda}\right)^2\right],$

$R_r = P_r/(\frac{1}{2}I_0^2) = 20\pi^2\left(\frac{2h}{\lambda}\right)^2.$

c) $D = \dfrac{4\pi|E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi}|E_\theta(\theta)|^2\sin\theta\, d\theta\, d\phi} = \dfrac{2}{\int_0^{\pi}\sin^3\theta\, d\theta} = 1.5 \longrightarrow 10\log_{10}D = 1.76\,(\text{dB}).$

P.10-7 $f = 180\times10^3\,(\text{Hz}) \longrightarrow \lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{180\times10^3} = 1{,}667\,(\text{m}).$

$h = 40\,(\text{m}) \ll \lambda$, with triangular current distribution.

a) From Problem P.10-6, we have, $(\beta = \frac{2\pi}{\lambda})$

$$|E_\theta|_{max} = \frac{30\beta h}{R}I_0 = \frac{30\times2\pi\times40}{1667\times160}\times100 = 2.83\,(\text{mV/m}).$$

$$|H_\phi|_{max} = \frac{1}{\eta_0}|E_\theta|_{max} = \frac{2.83\times10^{-3}}{377} = 7.51\times10^{-6}\,(\text{A/m}) = 7.51\,(\mu\text{A/m}).$$

b) $P_r = \frac{1}{2}\int_0^{\pi/2}\frac{|E_\theta|^2}{\eta_0}2\pi R^2\sin\theta\, d\theta = \frac{30^2}{120}(\beta hI_0)^2\int_0^{\pi/2}\sin^3\theta\, d\theta$

$$= \frac{30}{4}(\beta hI_0)^2\left(\frac{2}{3}\right) = 5(\beta hI_0)^2 = \frac{I_0^2}{2}\left[40\pi^2\left(\frac{h}{\lambda}\right)^2\right]$$

$$= \frac{100^2}{2}\times40\pi^2\left(\frac{40}{1667}\right)^2 = 1{,}136.5\,(\text{W}) \cong 1.14\,(\text{kW}).$$

c) $R_r = 2P_r/I_0^2 = 0.227\,(\Omega).$

P. 10-9 a) E-plane pattern function for Hertzian dipole is, from Eq. (10-10),

$$F_a(\theta) = \sin\theta.$$

Max. $F_a(\theta) = 1$ at $\theta_0 = 90°$.

Half-power points at $F_a(\theta_1) = F_a(\theta_2) = \frac{1}{\sqrt{2}}$.

$\longrightarrow \theta_1 = 45°, \quad \theta_2 = 135°.$

$\longrightarrow$ Beamwidth $\Delta\theta = \theta_2 - \theta_1 = 90°.$

b) E-plane pattern function for half-wave dipole is, from Eq. (10-38),

$$F_b(\theta) = \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}.$$

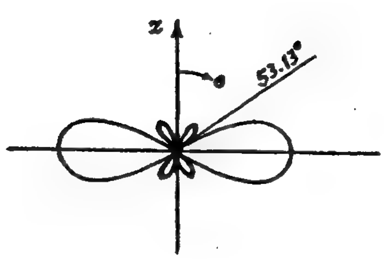
Max. $F_b(\theta) = 1$ at $\theta_0 = 90°$.

Half-power points at $F_b(\theta_1') = F_b(\theta_2') = \frac{1}{\sqrt{2}}$.

$\longrightarrow$ Beamwidth $\Delta\theta' = \theta_2' - \theta_1' = 129° - 51° = 78°.$

P. 10-10 Use Eq. (10-37):

$$F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}.$$



For $2h = 1.25\lambda$,

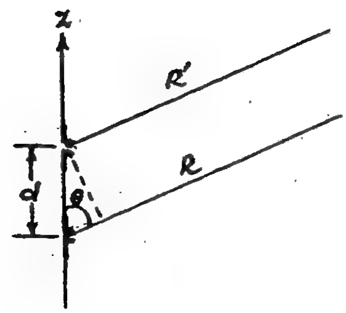
$$|F(\theta)| = \left|\frac{\cos(1.25\pi\cos\theta) - \cos(1.25\pi)}{\sin\theta}\right|$$

Width of main beam between the first nulls $= 2 \times 53.13° = 106.26°.$

P.10-11 Use Eq. (10-10) for Hertzian dipoles.

$$E_\theta = E_1(\theta) + E_2(\theta)$$
$$= \frac{jI(2h)}{4\pi}\eta_0\beta\sin\theta\left(\frac{e^{-j\beta R}}{R} + \frac{e^{-j\beta R'}}{R'}\right).$$

In the far zone, $R' \cong R - d\cos\theta$.

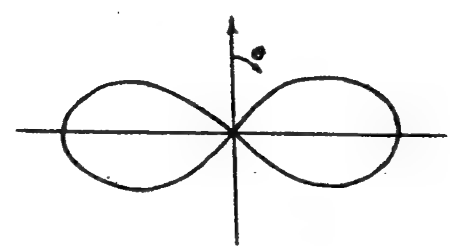


a) $$E_\theta \cong \frac{jI(2h)}{4\pi R}\eta_0\beta\sin\theta\cdot e^{-j\beta R}\cdot\left(1+e^{j\beta d\cos\theta}\right)$$
$$= \frac{j60Ih}{R}2\beta e^{-j\beta(R-\frac{d}{2}\cos\theta)}F(\theta),$$

where $F(\theta) = \sin\theta\cos\left(\frac{\beta d}{2}\cos\theta\right)$.

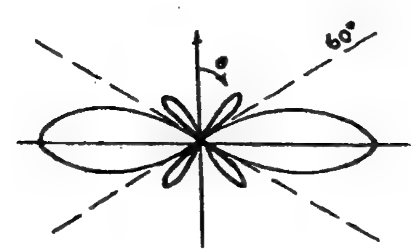
b) $d = \lambda/2$,

$$|F(\theta)| = \left|\sin\theta\cos\left(\frac{\pi}{2}\cos\theta\right)\right|.$$



c) $d = \lambda$,

$$|F(\theta)| = \left|\sin\theta\cos(\pi\cos\theta)\right|.$$



P.10-12 For an array of identical elements spaced a distance d apart, we have, from Eq. (10-54),

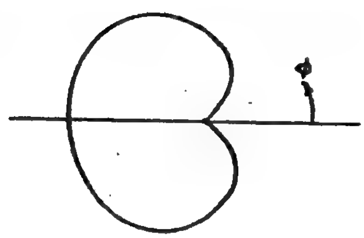
$$|E| = \frac{2E_m}{R_0}|F(\theta,\phi)|\left|\cos\frac{\psi}{2}\right|,$$

where $\psi = \beta d \sin\theta\cos\phi + \xi$.

In the H-plane of a dipole: $\theta = \pi/2$, $F(\frac{\pi}{2},\phi) = 1$.

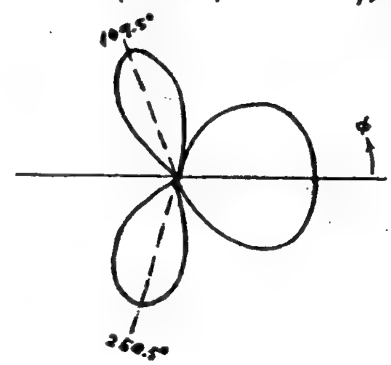
a) $d = \frac{\lambda}{4}$, $\xi = \frac{\pi}{2}$.

$|A(\phi)| = \left|\cos\frac{\psi}{2}\right| = \left|\cos\left[\frac{\pi}{4}(1+\cos\phi)\right]\right|$.



b) $d = \frac{3\lambda}{4}$, $\xi = \frac{\pi}{2}$.

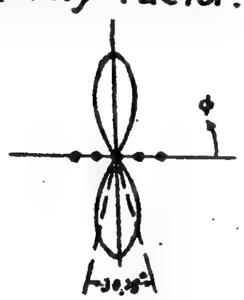
$|A(\phi)| = \left|\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\pi}{4}\right)\right|$.



P.10-13 Five-element broadside binomial array.

a) Relative excitation amplitudes: 1:4:6:4:1.

b) Array factor: $|A(\phi)| = \left|\cos\left(\frac{\pi}{2}\cos\phi\right)\right|^4$.



c) $\cos\left(\frac{\pi}{2}\cos\phi\right) = (\sqrt{2})^{-1/4}$

$\longrightarrow \phi = 74.86°$.

Half-power beamwidth

$= 2(90° - 74.86°)$

$= 30.28°$.

For uniform array, from Eq. (11-89):

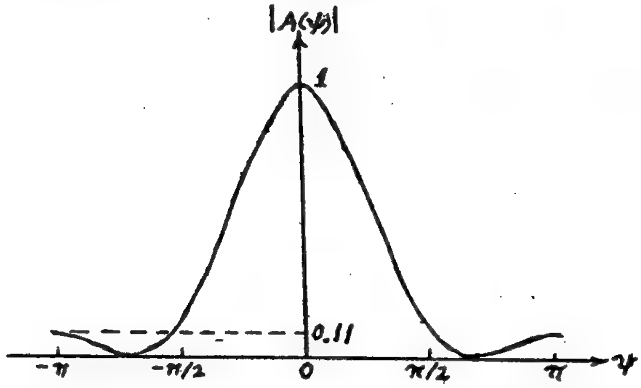
$$\frac{1}{5}\left|\frac{\sin(\frac{5\pi}{2}\cos\phi)}{\sin(\frac{\pi}{2}\cos\phi)}\right| = \frac{1}{\sqrt{2}} \longrightarrow \phi = 79.61°$$

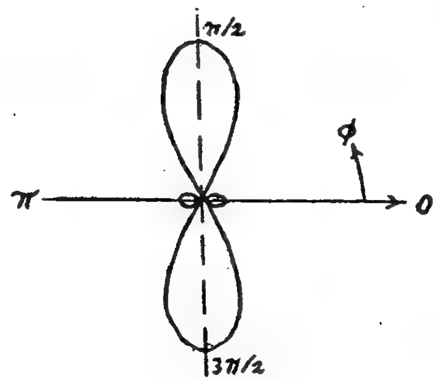
Half-power beamwidth for 5-element uniform array with $\lambda/2$ spacing $= 2(90° - 79.61°) = 20.78°$.

P.10-14 The normalized array factor of the five-element tapered array is

$$|A(\psi)| = \frac{1}{9}\left|1+2e^{j\psi}+3e^{j2\psi}+2e^{j3\psi}+e^{j4\psi}\right|$$
$$= \frac{1}{9}\left|e^{j2\psi}[3+2(e^{j\psi}+e^{-j\psi})+(e^{j2\psi}+e^{-j2\psi})]\right|$$
$$= \frac{1}{9}\left|3+4\cos\psi+2\cos 2\psi\right|.$$

A graph of $|A(\psi)|$ vs. ψ is shown below on the left.





For broadside operation: $\xi = 0$, $\psi = \beta d\cos\phi = \pi\cos\phi$.

$$|A(\phi)| = \frac{1}{9}\left|3+4\cos(\pi\cos\phi)+2\cos(2\pi\cos\phi)\right|.$$

This is plotted above on the right. The first sidelobe level is 0.11, or $20\log_{10}(1/0.11) = 19.2$ (dB) down from the main-beam radiation. This compares with 0.25, or 12 (dB) down for the five-element uniform broadside array shown in Fig. 10-11.

P.10-15 From Eqs. (10-39) and (10-60):

$$|E_\theta| = \frac{j60I_m N_1N_2}{R}e^{-j\beta R}\left|\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}A_x(\psi_x)A_y(\psi_y)\right|$$

where $A_x(\psi_x) = \frac{1}{N_1}\frac{\sin(N_1\psi_x/2)}{\sin(\psi_x/2)}$, $\quad \psi_x = \frac{\beta d_1}{2}\sin\theta\cos\phi$;

$A_y(\psi_y) = \frac{1}{N_2}\frac{\sin(N_2\psi_y/2)}{\sin(\psi_y/2)}$, $\quad \psi_y = \frac{\beta d_2}{2}\sin\theta\cos\phi$.

$$|F(\theta,\phi)| = \frac{1}{N_1N_2}\left|\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\frac{\sin(\frac{N_1\psi_x}{2})\sin(\frac{N_2\psi_y}{2})}{\sin(\frac{\psi_x}{2})\sin(\frac{\psi_y}{2})}\right|.$$

P. 10-16 $\quad \ell_e = \frac{1}{I(0)}\int_{-h}^{h} I(z)\,dz .$

a) Hertzian dipole of length $d\ell$.

$$I(z) = I(0), \quad h = \frac{1}{2}d\ell, \quad \sin(\beta\frac{d\ell}{2}) \cong \beta\frac{d\ell}{2}.$$

$$\ell_e = \int_{-d\ell/2}^{d\ell/2} \cos\beta z\, dz = d\ell.$$

b) Half-wave dipole with $h = \lambda/4$ and $I(z) = I(0)\cos\beta z$.

$$\ell_e = \int_{-\lambda/4}^{\lambda/4} \cos\beta z\, dz = \frac{2}{\beta}\sin(\beta\frac{\lambda}{4}) = \frac{2}{\beta} = \frac{\lambda}{\pi}.$$

c) Half-wave dipole with $h = \lambda/4$ and $I(z) = I(0)(1-4|z|/\lambda)$.

$$\ell_e = \int_{-\lambda/4}^{\lambda/4}(1-4|z|/\lambda)\,dz = 2\int_0^{\lambda/4}(1-4z/\lambda)\,dz = \frac{\lambda}{4}.$$

P. 10-17 $\quad \lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1\ (\text{m}).$

Half-wave dipole with sinusoidal current distribution

$$I(z) = I(0)\sin\beta(\frac{\lambda}{4}-|z|) \qquad (\beta\frac{\lambda}{4} = \frac{\pi}{2})$$
$$= I(0)\cos\beta z.$$

From Eq. (10-85) and problem P. 10-16 (b), $\ell_e = \frac{\lambda}{\pi}$.

From Eq. (10-35), we have, for $\theta = \pi/2$,

$$|E_i| = \frac{I(0)\eta_0\beta}{4\pi R}\ell_e = \frac{60}{\lambda R}I(0).$$

$$P_r = \frac{1}{2}I^2(0)R_r \longrightarrow I(0) = \sqrt{\frac{2P_r}{R_r}} = \sqrt{\frac{2\times2000}{73.1}} = 7.40\,(\text{A}).$$

$$|E_i| = \frac{60\times7.40}{1\times150} = 2.96\ (\text{V/m}).$$

a) $|V_{oc}| = |E_i\ell_e| = 2.96\times\frac{1}{\pi} = 0.942\ (\text{V}).$

b) For matched load,

$$P_L = \frac{V_{oc}^2}{8R_r} = \frac{0.942^2}{8\times73.1} = 1.52\times10^{-3}(\text{W}) = 1.52\ (\text{mW}).$$

P. 10-18 Eq. (10-80): $P_L = \frac{D_1 D_2 \lambda^2}{(4\pi r)^2} P_t$.

$r = 150$ (m), $P_t = 2\times10^3$ (W), $\lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1$ (m).

a) Parallel half-wave dipoles: $D_1 = D_2 = 1.64$.

$$P_L = \frac{1.64\times1.64\times1^2}{(4\pi\times150)^2}\times2\times10^3 = 1.514\times10^{-3}\ (\text{W})$$
$$= 1.514\ (\text{mW}).$$

b) Parallel Hertzian dipoles: $D_1 = D_2 = 1.50$.

$$P_L = 1.514\times\left(\frac{1.50}{1.64}\right)^2 = 1.267\ (\text{mW}).$$

P. 10-19 From Eqs. (10-12) and (10-14):

$$G_D = \frac{4\pi U(\theta,\phi)}{P_r} = \frac{4\pi R^2 \mathcal{P}_{av}}{P_r}. \qquad (1)$$

Using Eqs. (10-40) and (10-42) in (1):

$$G_D(\theta) = 1.64\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2. \qquad (2)$$

a) Substituting (2) in Eq. (10-75):

$$A_e(\theta) = \frac{\lambda^2}{4\pi}G_D(\theta) = 0.13\lambda^2\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2.$$

b) Max. value of $A_e(\theta)$ for $f = 10^8$ (Hz), $\lambda = \frac{c}{f} = 3$ (m) occurs at $\theta = \frac{\pi}{2}$. $A_e(\frac{\pi}{2}) = 0.13\lambda^2 = 1.17$ (m^2).

c) Max. value of $A_e(\theta)$ for $f = 2\times10^8$ (Hz), $\lambda = 1.5$ (m):

$A_e(\frac{\pi}{2}) = 0.13\times1.5^2 = 0.29$ (m^2),

which is smaller than $A_e(\frac{\pi}{2})$ for $f = 10^8$ (Hz) because the wavelength is shorter at $f = 2\times10^8$ (Hz).

P. 10-20 Antenna gain: $10\log_{10} G_D = 20$ (dB)

$$\longrightarrow G_D = 100.$$

$$f = 3\times 10^9\ (\text{Hz}) \longrightarrow \lambda = \frac{c}{f} = \frac{3\times 10^8}{3\times 10^9} = 0.1\ (\text{m}).$$

a) Power density at target, $\mathcal{P}_T = \frac{P_t}{4\pi r^2} G_D$.

$$\mathcal{P}_T = \frac{E_T^2}{2\eta_0} = \frac{120\times 10^3\times 100}{4\pi(8\times 10^3)^2} = 0.0149\ (\text{W/m}^2)$$

$$\longrightarrow E_T = \sqrt{0.0149\times 2\times 377} = 3.35\ (\text{V/m}).$$

b) Power intercepted by target $= \sigma_{bs}\mathcal{P}_T = 15\times 0.0149 = 0.224$ (W).

c) Scattered power density at radar antenna

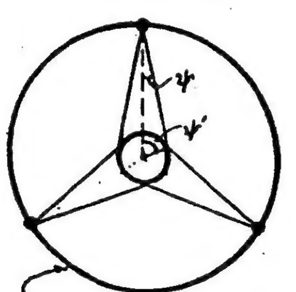
$$\mathcal{P}_s = \frac{\sigma_{bs}\mathcal{P}_T}{4\pi r^2} = \frac{0.224}{4\pi(8\times 10^3)^2} = 2.78\times 10^{-10}\ (\text{W})$$

Reflected power absorbed by antenna $= \mathcal{P}_s A_e$

$$= 2.78\times 10^{-10}\left(\frac{\lambda^2}{4\pi}G_D\right) = 2.78\times 10^{-10}\left(\frac{0.1^2}{4\pi}\times 100\right) = 22.1\times 10^{-12}\ (\text{W})$$
$$= 22.1\ (\text{pW}).$$

P. 10-21 Earth radius $= 6{,}380$ (km).

Altitude of geosynchronous satellites $= 36{,}500$ (km)



Geosynchronous orbit

Geosynchrous orbit radius $= 6{,}380 + 36{,}500$
$= 42{,}880$ (km).

$$\psi = \sin^{-1}\left(\frac{6{,}380}{42{,}880}\right) = 8.56°$$
$$\psi' = 90° - 8.56° = 81.44°$$

a) Two satellites cover only $2\times(2\psi') = 326° < 360°$.

Use three satellites in equatorial plane: $3\times(2\psi') = 489° > 360°$.

Polar regions are not covered because $\psi' < 90°$.

b) Let P_t = Power transmitted by satellite antenna.

$\mathcal{P}_c$ = Power density within the cone $= \frac{G_D}{4\pi r^2} P_t$.

Area of cone cap on earth $= \int_0^{2\pi}\int_0^{\psi} r^2 \sin\theta\, d\theta\, d\phi$

$$= 2\pi r^2(1-\cos\psi) \cong 2\pi r^2(\psi^2/2) = \pi(r\psi)^2.$$

$$\therefore P_t = \pi(r\psi)^2\mathcal{P}_c \longrightarrow \psi = \frac{1}{r}\sqrt{P_t/\pi\mathcal{P}_c} = 2/\sqrt{G_D} \longrightarrow \text{Main-lobe beamwidth} = 2\psi = \frac{4}{\sqrt{G_D}}.$$

P.10-22 a) From Eq. (10-80):

$$P_t = \frac{(4\pi r)^2}{G_e G_s \lambda_s^2} P_L ,$$

where the subscripts e and s denote earth and Satellite respectively.

$$\lambda_e = \frac{3 \times 10^8}{14 \times 10^9} = 2.14 \times 10^{-2} \text{ (m)},$$

$$G_e = 10^{(55/10)} = 3.16 \times 10^5;$$

$$\lambda_s = \frac{3 \times 10^8}{12 \times 10^9} = 2.50 \times 10^{-2} \text{ (m)},$$

$$G_s = 10^{(35/10)} = 3.16 \times 10^3.$$

$$r = 3.65 \times 10^7 \text{ (m)}, \quad P_L = 8 \times 10^{-12} \text{ (W)}.$$

$$\longrightarrow P_t = 2.7 \text{ (W)}.$$

b) From Eq. (10-84): $P_t = \frac{4\pi}{\sigma_{bs}} \left(\frac{\lambda_e r^2}{A_e}\right)^2 P_L$

$$A_e = \frac{\lambda_e^2}{4\pi} G_e = \frac{(2.14 \times 10^{-2})^2}{4\pi} \times 3.16 \times 10^5$$

$$= 11.5 \text{ (m}^2\text{)}.$$

$$P_t = \frac{4\pi}{25}\left(\frac{2.14 \times 10^{-2} \times 3.65^2 \times 10^{14}}{11.5}\right)^2 \times 0.5 \times 10^{-12}$$

$$= 1.54 \times 10^{12} \text{ (W)} = 1.54 \text{ (TW)}.$$